Budapest University of Technology and Economics

Department of Mechanics, Materials and Structures

English courses

Reinforced Concrete Structures

Code: BMEEPSTK601

Lecture no. 6:

SHEAR AND TORSION

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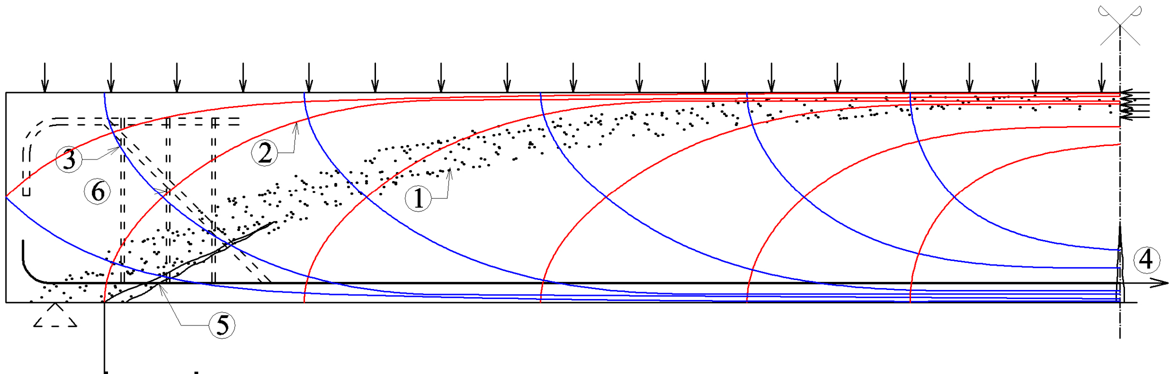
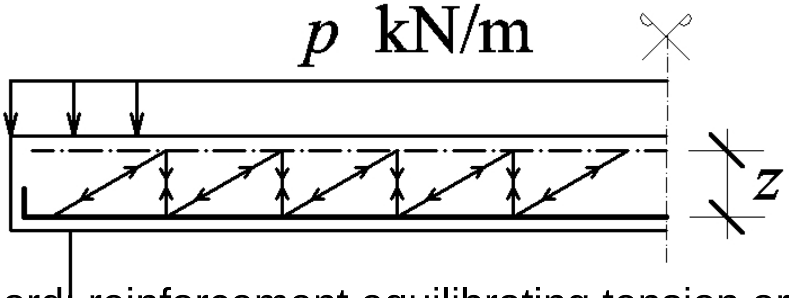
I. Shear

1. Ways of modeling shear transfer in rc beams



1: tha vault action
2: compression trajectories
3: tension trajectories
4: 1st shear crack, tension in the bottom reinforcement
5: 1st shear crack
6: elements of the shear reinforcement crossing the shear crack: links and bent-up bars

The truss model of Mörsch showing the way of transmitting shear to the support of simple supported rc beams



-lower chord: reinforcement equilibrating tension originated by flexure

-on top: concrete compression chord

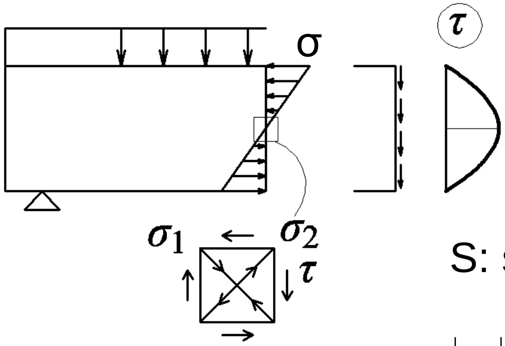
-compressed concrete struts, with inclination angle θ

-vertical tie-up forces absorbed by links

In the following the concrete compression strut inclination angle θ=45º is considered for convenience in manual calculations.

In EC2 $1 \leq \cot\theta \leq 2.5$ is allowed, that is: $21{,}6^\circ \leq \theta \leq 45^\circ$

2. Absorbing shear in uncracked state



$$\tau = \frac{SV}{bI_x}$$

S: static moment V:shear force

$$|\sigma_1| = |\sigma_2| = |\tau|$$

strength rates: $f_{ct,d}$=0,1 unit f_{cd}=1 unit τ_d≈0,15 unit

consequence: tension failure occurs first: cracking parallel to σ_2

Approximate shear resistance of the concrete section:

$V_{Rd,c} = c b_w d f_{ct,d}$ *c* tabulated in DA

Values of *c* for concrete grade C20/25

Values of c for calculation of $V_{Rd,c} = c\, b_w\, d\, f_{ctd}$, concrete: C20/25										
	ρ_l [%]	d [mm]								
		≤ 200	300	400	500	600	700	800	900	1000
C20/25	0,00	0,429	0,371	0,338	0,316	0,301	0,288	0,279	0,271	0,264
f_{ctd} =1,0	0,25	0,429	0,371	0,340	0,325	0,314	0,305	0,298	0,293	0,288
	0,50	0,501	0,455	0,428	0,409	0,395	0,385	0,376	0,369	0,363
	1,00	0,632	0,574	0,539	0,515	0,498	0,485	0,474	0,465	0,457
	2,00	0,796	0,723	0,679	0,649	0,628	0,611	0,597	0,585	0,576

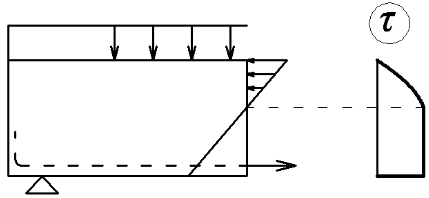
3. Ways of absorbing shear in cracked state

Neglected components:

-shear strength of the compression zone

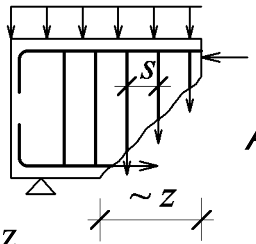
-shear absorbed by friction along the shear cracks

-dowel action of bars of the tension reinf.

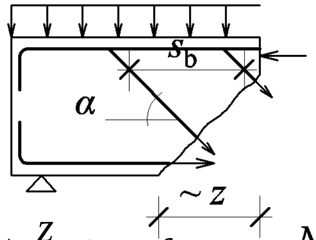


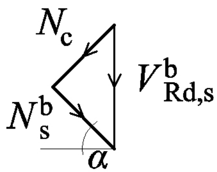
Shear equilibrated by links and bent-up bars:

$$\tau = \frac{V}{bz}$$



A_{sw}: area of two legs!





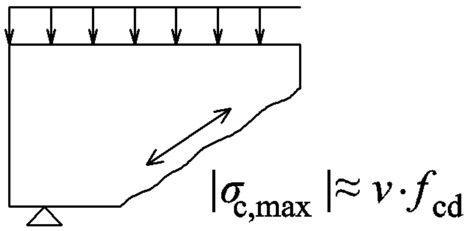
$$V_{Rd,s} = \frac{z}{s} A_{sw} f_{ywd}$$

$$V^b_{Rd,s} = (\sin\alpha + \cos\alpha)\frac{z}{s_b} A_{sw,b} f_{ywd}$$

for α=45º:

$$V^b_{Rd,s} = \sqrt{2}\,\frac{z}{s_b} A_{sw,b} f_{ywd}$$

4. The maximum shear capacity limited by the compression strength of the concrete



Based on test results (in case of applying vertical links):

$$V_{Rd,\max} = 0{,}5 b_w z \nu f_{cd}$$

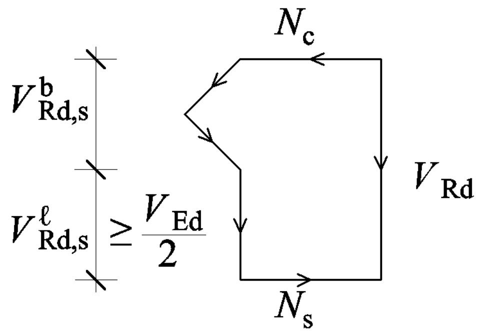
for vertical links + bent-up bars: 0,5→0,75

z≈0,9d can be substituted

$\nu = 0.6\left(1 - \frac{f_{ck}}{250}\right)$ effectiveness factor

5. Design condition of the shear capacity

$$V_{Rd} = \min \begin{Bmatrix} V_{Rd,\max} \\ \max \begin{Bmatrix} V_{Rd,c} \\ V_{Rd,s} = V_{Rd,s}^{\ell} + V_{Rd,s}^{b} \end{Bmatrix} \end{Bmatrix} \geq V_{Ed}$$



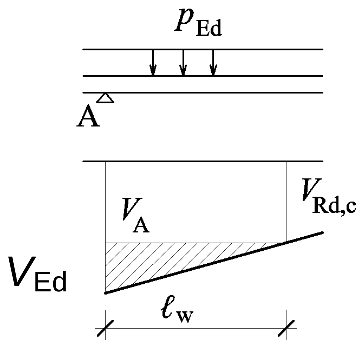
it is to be respected that: $V_{Rd,s}^{\ell} \geq 0{,}5 V_{Ed}$

6. The practical way of shear design

If $V_{Ed} \geq V_{Rd,c}$

and no bent-up bars are used, set diameter of vertical links, and calculate the necessary spacing of links:

$$s_s = \frac{0{,}9 d A_{sw} f_{ywd}}{V_{Ed}}$$



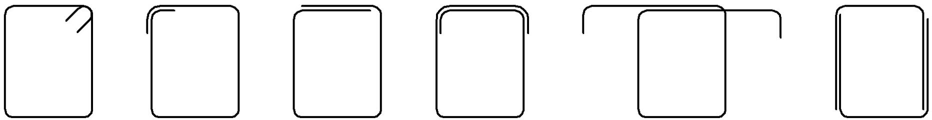
if bent-up bars are used:

$$s_s = \frac{0{,}9 d A_{sw} f_{ywd}}{V_{Ed} - V^b_{Rd,s}}$$

Dashed area: shear to be equilibrated by shear reinforcement

where $V_{Ed} < V_{Rd,c}$ min. links can be used (see DA)

Different links:

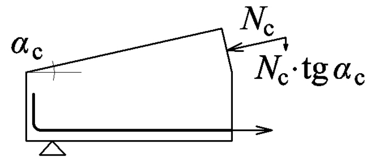


7. Special poblems in shear design

Variable height of the beam

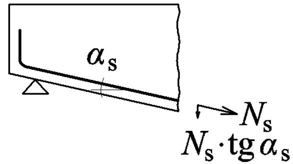
a) variation on side of the compression zone

$$V'_{Ed} = V_{Ed} - N_c \tan \alpha$$



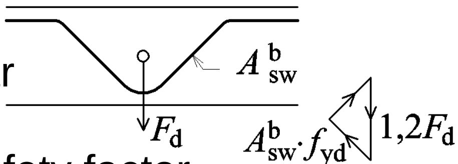
b) variation on side of the tension zone

$$V'_{Ed} = V_{Ed} - N_s \tan \alpha_s$$



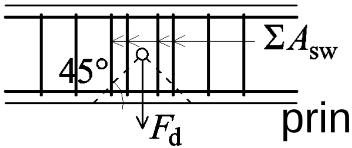
Tieing-up concentrated load or secundary beam

by bent-up bar

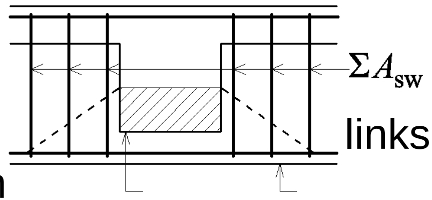


1,2: local safety factor

by vert. links



links of principal beam

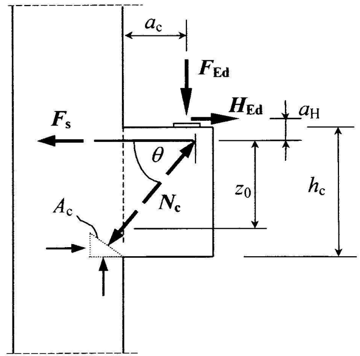


8. The short cantilever

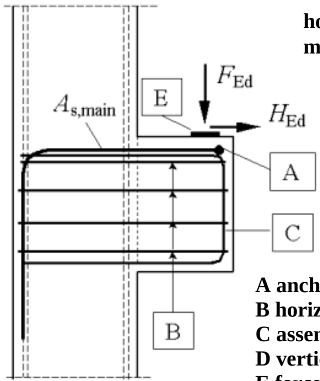
Force to equilibrate by the main bars:

$$F_s \cong F_{Ed}\left(\frac{a_c}{z_0}+0{,}1\right)$$

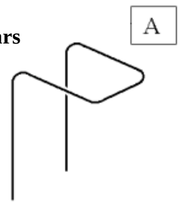
$$45^\circ \leq \theta \leq 68^\circ$$

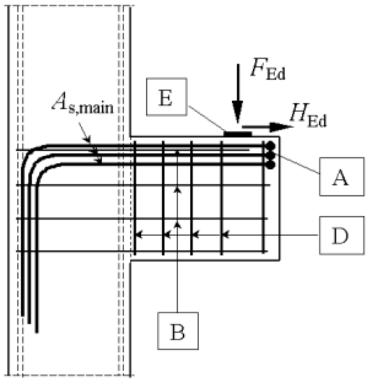


column



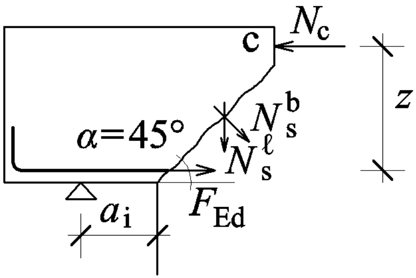
hooked
main bars





A anchorage device hook
B horizontal closed links
C assembly bar
D vertical closed links
E force transmitting member

9. Check of the beam end



The force to be absorbed by the tension reinforcement at the beam end:

$$\Sigma M_c = 0: \rightarrow F_{Ed}$$

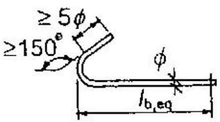
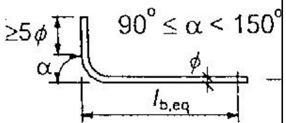
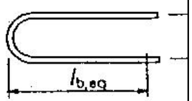
Formulae for F_{Ed} see in DA:

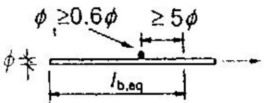
Values of the tensile force F_{Ed}		
There is no designed shear reinforcement	There is designed shear reinforcement	
	. θ = 45 crack inclination angle	
	links	bent-up bars also
$\left(1.1+1.1\frac{a_i}{d}\right)\lvert V_{Ed}\rvert$	$\left(0.5+1.1\frac{a_i}{d}\right)\lvert V_{Ed}\rvert$	$\left(0.25+1.1\frac{a_i}{d}\right)\lvert V_{Ed}\rvert$
Approximation: ≈*h*/2≈(*d*/0.85)/2:		
$1.75\lvert V_{Ed}\rvert$	$1.15\lvert V_{Ed}\rvert$	$0.9\lvert V_{Ed}\rvert$

10. Reduction of the anchorage length by 90º bents and hooks

Anchorage of tension bars at the beam end is problematic due to lack of space. Solution: use of hooks, bends, loops, welded anchorage devices.

$$\ell_{b,red} = \alpha_a \ell_b$$

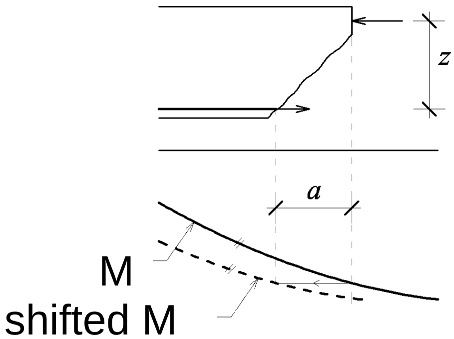
Hook	90º bend	Loop	α_a
			0,7[1]
welded transverse bar within l_{bd}			0,7



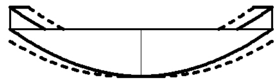
[1] **The reduction is valid only if along the bent portion the concrete cover in direction perpendicular to the plane of bending is $>3\phi$, transverse compression is acting, and links are used, otherwise α_a= 1,0.**

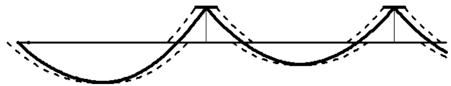
11. Parallel shifting of the moment diagr am due to diagonal shear cracks

Reason of shifting: inclination of shear cracks



Direction of shifting M:





Extent of shifting: $a = \begin{cases} z = 0{,}9d & \text{no shear reinf.} \\ 0{,}5z = 0{,}45d & \text{shear reinforcement: links} \\ 0{,}25z = 0{,}225d & \text{shear reinf.: links+bent-up bars} \end{cases}$

12. Constructional rules of links and bent-up bars

At least half of the shear force should be equilibrated by links.
The shear steel ratio: $\rho_w = A_{sw}/(s \cdot b_w \cdot \sin\alpha)$

$\rho_{w,min} = (0{,}08\sqrt{f_{ck}})/f_{yk}$ is tabulated in DA:

Values of the minimum shear steel ratio: $\rho_{w,min}$ (%o)									
f_{yk}	concrete								
	C12/16	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/67
500	1.00	1.00	1.00	1.00	1.00	1.00	1,01	1,07	1,13
400	0,69	0,80	0,89	1,00	1,10	1,18	1,26	1,34	1,41
240	1,15	1,33	1,48	1,67	1,81	1,95	2,05	2,21	2,33

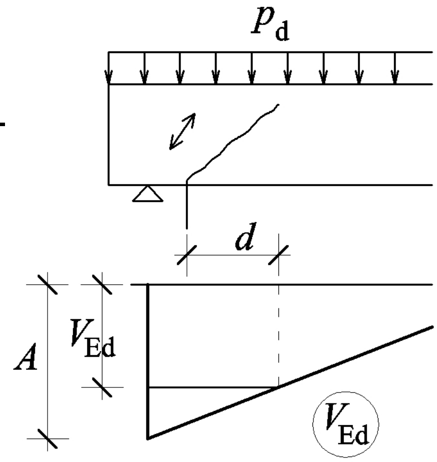
Maximum spacing of links $s_{l,max} = 0{,}75d$

In case of designed compression steel $s_l \leq 15\phi'$

Maximum spacing of 45° bent-up bars $s_{b,max} = 1{,}2d$

13. Shear transmitted by diagonal compression to the support

This is possible according to EC2, but will be neglected for simplification as a safe approximation

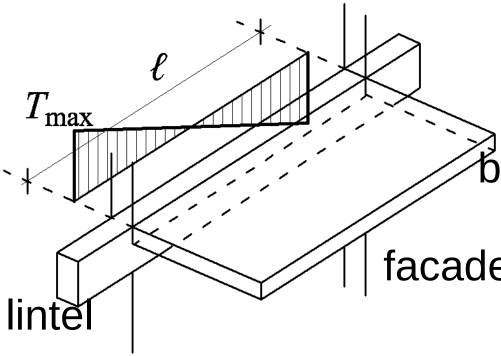


II. Torsion

1. Way of handling of torsion in design practice

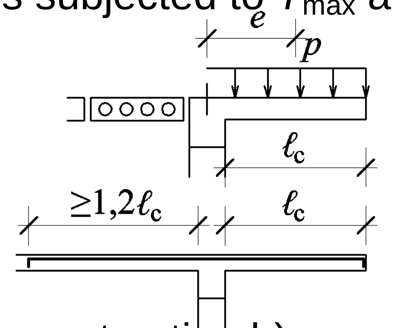
Try to avoid torsion if possible!

Example $$T_{\max} = \frac{pl_c el}{2}$$



Section of construction a)

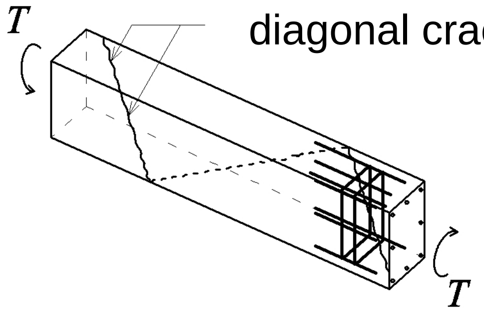
The lintel is subjected to $T_{\max}$ at the support



Section of construction b)

Moments of the balcony slab are equilibrated by the joining inside monolithic rc slab. The lintel is not subjected to torsion!

2. The behaviour of rc beams subjected to torsion

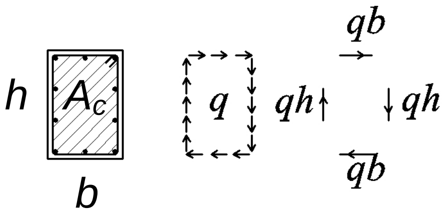


diagonal cracking continuing along all the four sides

Both logitudinal bars and links are intersecting the cracks: they both work in equilibrating torsion

Due to diagonal cracking the rigidity of the member (beam) is much reduced. The resistance to flexural deformations is decreasing significantly by the effect of torsion.

3. The shear flow equilibrating torsion along the perimeter of the section



q (kN/m)

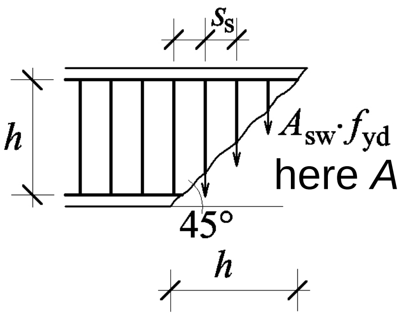
(h and b are measured here between centerlines of links)

The torsion produced by the shear flow q is:

$$T = 2qhb = q2A_c$$

Let us express the shear flow q by capacities of links and longitudinal bars!

4. Torsional moment capacity due to links



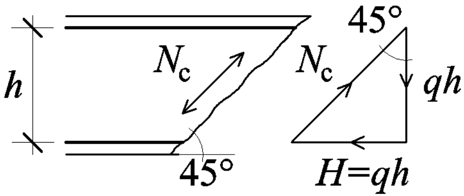
here A_{sw} stands for area of one leg of the links!

$$qh = nA_{sw} f_{ywd} = \frac{h}{s_s} A_{sw} f_{ywd} \rightarrow \quad q = \frac{A_{sw} f_{ywd}}{s_s}$$

The *torsional moment capacity due to links* by substituting the : expression obtained for *q*:

$$T_{Rd,s} = \frac{A_{sw} f_{ywd}}{s_s} 2A_c$$

5. Torsional moment capacity due to longitudinal bars



The tensile force to be equilibrated by longitudinal bars:

$$\Sigma H = 2qh + 2qb = qp_c = A_{sl} f_{yd}$$

here 2*h* + 2*b*= p_c is the perimeter measured along centerline of links

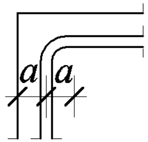
Expressing *q:* $q = \dfrac{A_{sl} f_{yd}}{p_c}$, and substituting *q* in *T*, the *torsional moment capacity due to* longitudinal bars*:*

$$T_{Rd,A_{sl}} = \frac{A_{sl} f_{yd}}{p_c} 2A_c$$

6. The torsional moment capacity of rc beams

$$T_{Rd} = Min\begin{Bmatrix} T_{Rd,s} \\ T_{Rd,A_{sl}} \end{Bmatrix} \leq T_{Rd,\max} = \upsilon f_{cd} A_c t_{eff}$$

Here t_{eff}=max$\left[\frac{A_c}{p_c}, 2a \right]$



Uniformly distributed – closed - links and longitudinal bars should be designed, independently from links iand longitudinal bars designed for shear and moment.

Closed links to be designed for torsion