Budapest University of Technology and Economics

Department of Strength of Materials and Structures
English courses
Reinforced Concrete Structures
Code: BMETKEPB603

Lecture no. 3:

The uncracked (1st), cracked (2nd) and plastic (3rd) state of stresses

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2. Example of an rc beam subjected to flexure

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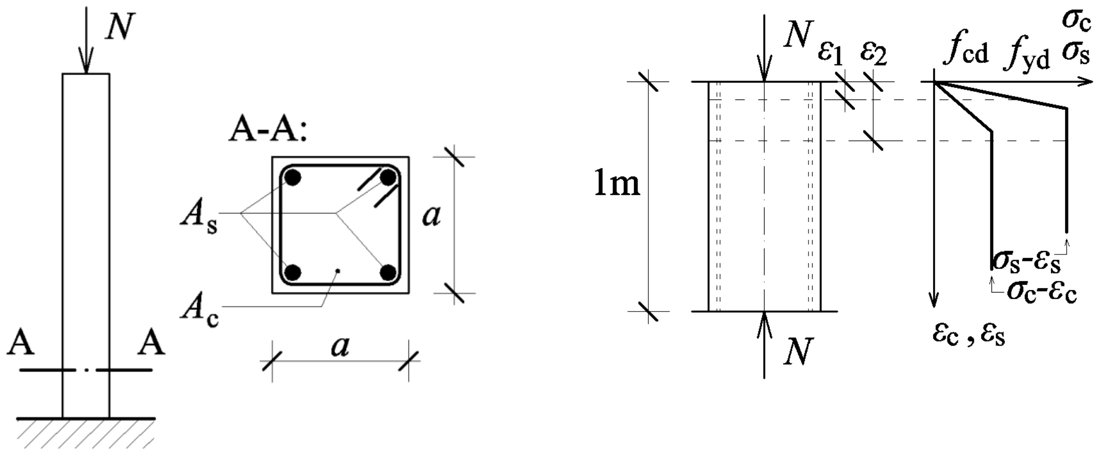
Plastic state

3. The moment-curvature relationship in different stress states
4. The resistance moment steel percentage relationship of rectangular sections
5. Numerical example

1. Example of an axially loaded rc column



Elastic state: $\varepsilon = \varepsilon_1 = \varepsilon_c = \frac{\sigma_c}{E_c} = \varepsilon_s = \frac{\sigma_s}{E_s} \rightarrow \quad \sigma_s = \frac{E_s}{E_c}\sigma_c = \alpha_c \sigma_c$

α_c: modular ratio E_s= 200 000 N/mm^2 $E_c \approx$10000 to 20000 N/mm^2

$\alpha_c \approx$ 10 to 20 (the concrete is 10 to 20 times softer than steel)

Equilibrium:

$\underline{\Sigma N = 0:}$ $\quad N = N_c + N_s = A_c\sigma_c + A_s\sigma_s = A_c\sigma_c + A_s\,\alpha_c\,\sigma_c = \sigma_c(A_c + \alpha_c\,A_s) =$

$= \sigma_c A_{i1}$

Introducing: $A_{i1} = A_c + \alpha_c\,A_s$ idealized area of the rc section in uncracked state:

$$\sigma_c = \frac{N}{A_{i1}} \leq f_{cd}$$

$$\sigma_s = \alpha_c \sigma_c \leq f_{yd}$$

Increasing the force N to cause a specific deformation

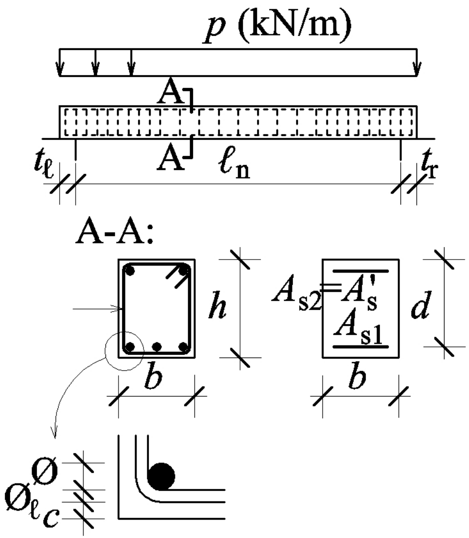
$$\varepsilon = \varepsilon_2 > \varepsilon_{c1} = \frac{f_{cd}}{E_c} \quad \text{and} > \varepsilon_{s1} = \frac{f_{yd}}{E_s}$$

both concrete and steel are in the plastic state: $\sigma_c = f_{cd}$ and $\sigma_s = f_{yd}$

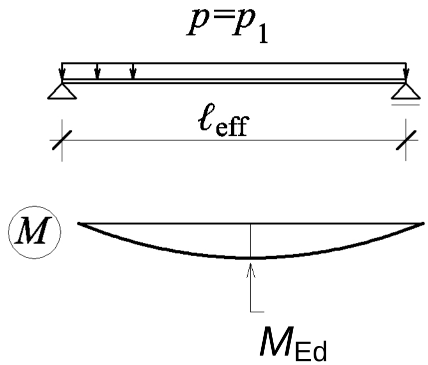
Equilibrium:

$$\underline{\Sigma N = 0:} \quad N = N_c + N_s = A_c f_{cd} + A_s f_{yd}$$

2. Example of an rc beam subjected to flexure



Model:

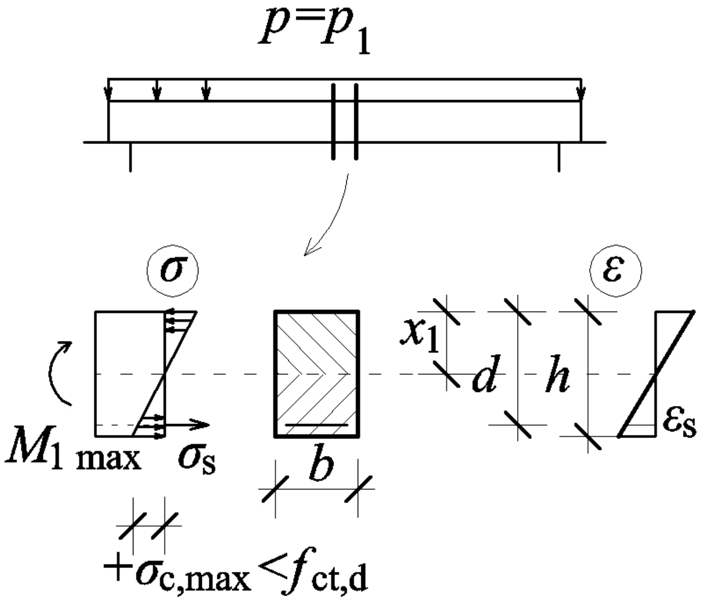


$$l_{eff} = l_n + \min\begin{Bmatrix} h/2 \\ t_l/2 \end{Bmatrix} + \min\begin{Bmatrix} h/2 \\ t_r/2 \end{Bmatrix}$$

$$M_{Ed} = \frac{p_d l_{eff}^2}{8}$$

By low load intensity p_1 the beam section will not crack at $M_{Ed}=M_{1max}$:

2.1 Elastic uncracked state



The force equilibrium ($\underline{\Sigma N = 0}$) of the section subjected to flexure is expressed by the condition, that the static moment is zero with respect the center of the idealized uncracked cross-section:

$$\underline{\Sigma S = 0}: \quad b x_1 \frac{x_1}{2} - b(h - x_1)\frac{h - x_1}{2} - \alpha_c A_s (d - x_1) = 0 \rightarrow \quad x_1$$

Inertia of the idealized cross-section in the uncracked state:

$$I_{i1} = \frac{b x_1^3}{3} + \frac{b(h - x_1)^3}{3} + \alpha_c A_s (d - x_1)^2$$

Stresses in uncracked state:

$$(-)\sigma_{c,max} = \frac{M_1}{I_{i1}} x_1 \leq f_{cd} \qquad +\sigma_{c,max} = \frac{M_1}{I_{i1}}(h - x_1) \leq f_{ct,d}$$

$$\sigma_s = \alpha_c \frac{M_1}{I_{i1}}(d - x_1) \leq f_{yd}$$

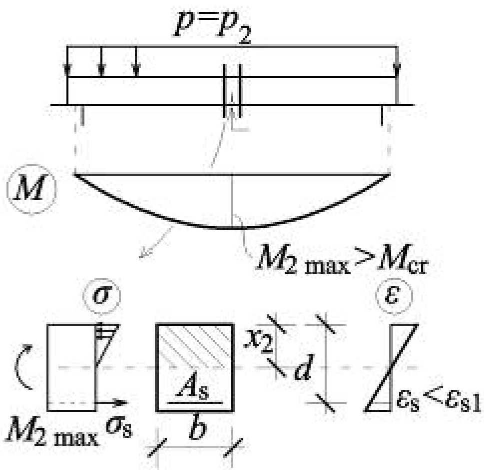
The moment causing cracking of the concrete is the one which produces $f_{ct,d}$ in the extreme tension fibre:

$$M_{cr} = f_{ct,d} \frac{I_{i1}}{x_1}$$

The cracking load: $p_{cr} = \frac{8M_{cr}}{l_{eff}^2}$

When further increasing the load intensity to $p_2 > p_{cr}$, crack formation at midspan can be observed

Elastic cracked state



Now the equilibrium of forces ($\Sigma N = 0$) is expressed by the condition, that the static moment of the cracked rc section is zero with respect to the neutral axis:

$$\Sigma S = 0: \qquad b x_2 \frac{x_2}{2} - \alpha_c A_s (d - x_2) = 0 \rightarrow \quad x_2$$

Inertia of the idealized rc section in cracked state:

$$I_{i2} = \frac{b x_2^3}{3} + \alpha_c A_s (d - x_2)^2$$

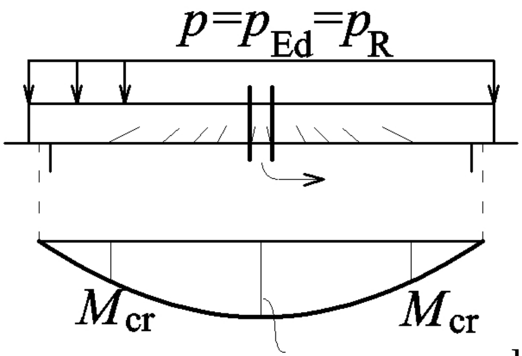
Stresses in cracked state:

$$(-)\sigma_{c,max} = \frac{M_2}{I_{i2}} x_2 \quad \leq f_{cd}$$

$$\sigma_s = \alpha_c \frac{M_2}{I_{i2}} (d - x_2) \quad \leq f_{yd}$$

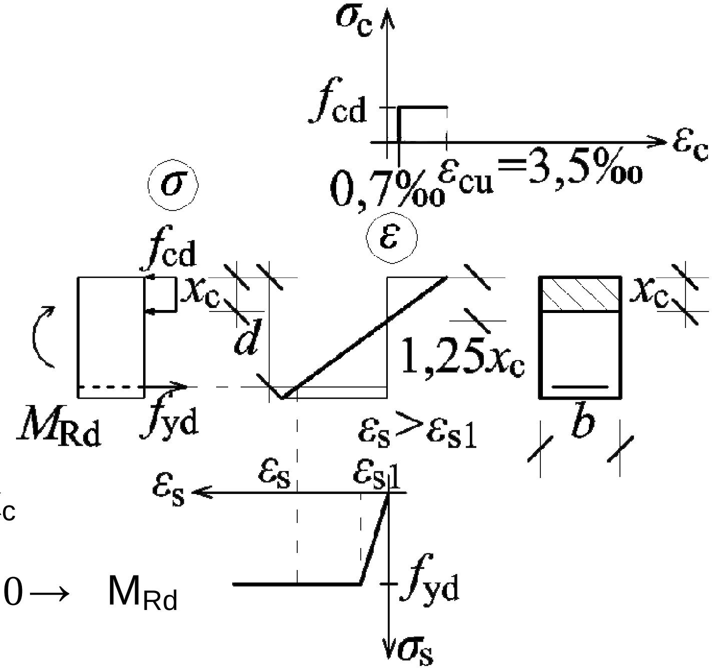
If we increase further the load intensity p, the the steel yields at M_{max} and the rupture of concrete at the extreme compression fibre means reaching the *plastic state*:

2.3 Plastic state



$$M_{Ed} = \frac{p_{Ed} l_{eff}^2}{8} = M_{Rd} \text{ (by design)}$$

Consider equilibrium of the part of the beam cut out by two parallel planes at M_{max}

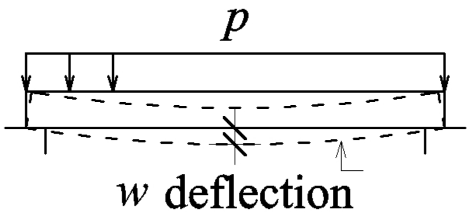


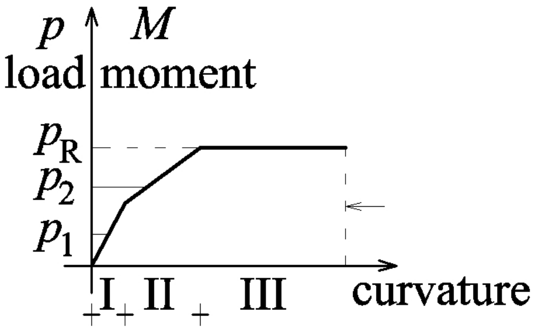
$\Sigma N = 0:$ $A_s f_{yd} - b x_c f_{cd} = 0 \rightarrow$ x_c

$\Sigma M_s = 0:$ $M_{Rd} - b x_c f_{cd} (d - \frac{x_c}{2}) = 0 \rightarrow$ M_{Rd}

where $(d - \frac{x_c}{2})$ =z is the internal level arm

Due to cracking the flexural rigidity of rc members will be reduced significantly. This can be observed on the moment-curvature (or load-deflection) diagram below.

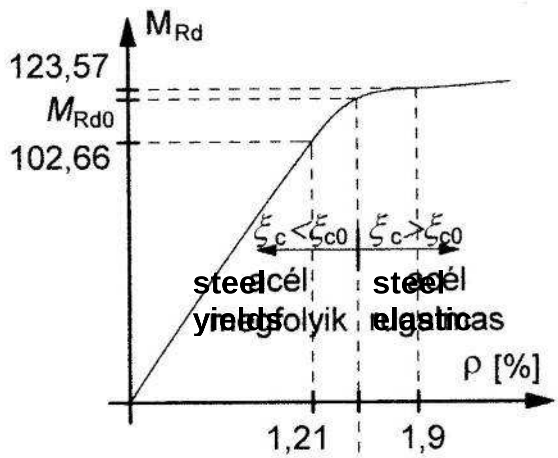




4.The resistance moment steel percentage relationship of rectangular, simply reinforced sections

$$M_{Rd,0} = b\xi_{c0} d f_{cd} (d - \frac{\xi_{c0} d}{2})$$

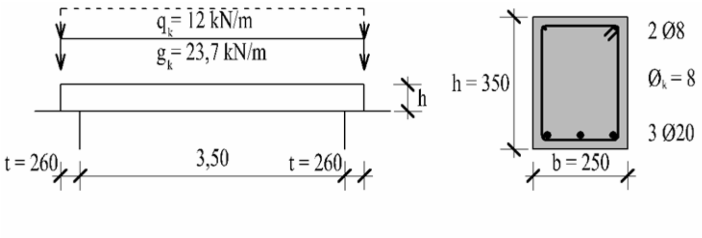
Recommendation:
design of doubly reinforced section (that is design comp-ression reinforcement) if
$M_{Ed} > M_{Rd,0}$



normally reinforced **over-reinforced**

5.Numerical example

Determine the cracking moment of the beam section investigated in the numerical example of the previous lecture!



C20/25-32/KK

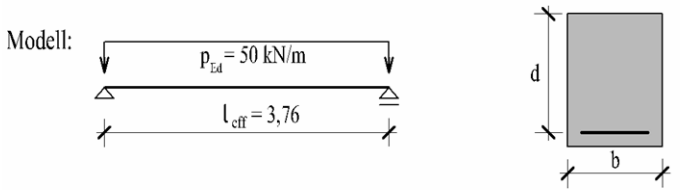
B 60.50

c_{nom}: 20 mm (conc. Cover

E_{cm}= 30000 N/mm^2

E_s= 200000N/mm^2

Some details of the calculations shown earlier to determine the resistance moment of the section:



$$l_{eff} = 3.5 + \min \left\{ \begin{array}{l} 2 \cdot \frac{0,35}{2} \\ 2 \cdot \frac{0,26}{2} \end{array} \right\} = 3.76 \text{ m}$$

Design load intensity:

$$p_{Ed} = \gamma_G g_k + \gamma_Q q_k = 1{,}35 \cdot 23.7 + 1{,}5 \cdot 12 = 50{,}0 \text{ kN/m}$$

$$M_{Ed} = p_{Ed} \frac{l^2_{eff}}{8} = 50 \cdot \frac{3{,}76^2}{8} = 88{,}36 \text{ kNm}$$

The top reinforcement will be neglected, the area of 3Ø20 on tension side:

$$A_s = 942 \text{ mm}^2 \text{ (VS. 8. o.)}$$

Effective depth:

$$d = 350 - 20 - 8 - 20/2 = 312 \text{ mm} \quad \dots \; x_c = 122 \text{ mm}$$

$$z = d - x_c/2 = 312 - 61 = 251 \text{ mm}$$

$M_{Rd} = A_s f_{yd} z = 942 \cdot 435 \cdot 251 = 102{,}66 \cdot 10^6$ Nmm = 102,66 kNm > M_{Ed} = 88,36 kNm the section is safe

To determine the cracking moment, tensile strength of the concrete is needed.

From DA table for concrete C20/25: $f_{ctd} = 1 \text{ N/mm}^2$

$$\underline{\Sigma S = 0}: \quad b x_1 \frac{x_1}{2} - b(h - x_1)\frac{h - x_1}{2} - \alpha_c A_s (d - x_1) = 0 \rightarrow \quad x_1$$

$$\alpha_c = \frac{E_s}{E_{cm}} = \frac{200000}{30000} = 6{,}67$$

$$250 \cdot x_1 \frac{x_1}{2} - 250 \cdot (350 - x_1) \cdot \frac{350 - x_1}{2} - 6{,}67 \cdot 942 \cdot (312 - x_1) = 0$$

x_1= 212,7 mm

$$I_{i1} = \frac{b x_1^3}{3} + \frac{b(h - x_1)^3}{3} + \alpha_c A_s (d - x_1)^2 = \frac{250 \cdot 212{,}7^3}{3} + \frac{250 \cdot (350 - 212{,}7)^3}{3} +$$

$$+ 6{,}67 \cdot 9{,}42 \cdot (312 - 212{,}7)^2 =$$

$$9{,}019 \cdot 10^8 + 2{,}157 \cdot 10^8 + 0{,}619 \cdot 10^8 = 11{,}795 \cdot 10^8 \text{mm}^4$$

$$M_{cr} = f_{ct,d} \frac{I_{i1}}{h - x_1} = 1{,}0 \cdot \frac{11{,}795 \cdot 10^8}{350 - 212{,}7} = 8{,}59 \cdot 10^6 \text{Nmm} = 8{,}59 \text{ kNm}$$

$$\frac{M_{cr}}{M_{Ed}} = \frac{8,59}{88,36} = 0,097$$

, that is the beam section will crack at 9,7% of the actual moment maximum, so that practically all the beam will be in cracked state.