Budapest University of Technology and Economics

Department of Strength of Materials and Structures
English courses
Reinforced Concrete Structures
Code: BMETKEPB603

Lecture no. 2:

Modelling, bending of rectangular sections

Content:

1. Modelling of an rc. beam, elements of the reinforcement system
2. Basic suppositions of the machanical behavior
3. The cross-section of an rc. beam, notations
4. Deformations and stresses of rc sections subjected to flexure, equilibrium conditions
5. Cases of cross-section check and design
6. Special cases: steel in elastic state at rupture of the concrete
7. The minimum quantity of the tensile reinforcement
8. Limitation of the compression force absorbed by the compress-ion reinforcement

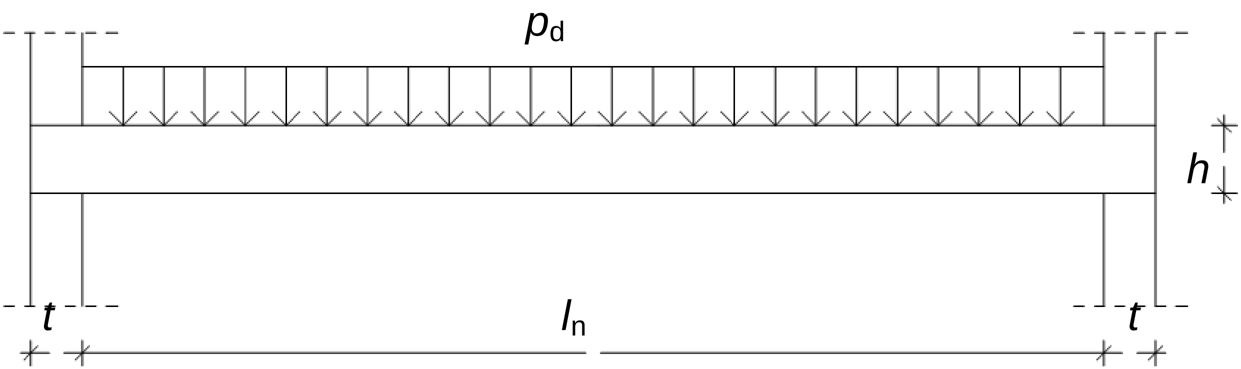
9 Principles of the realization of the necessary steel area

10.T-sections, flanged beams

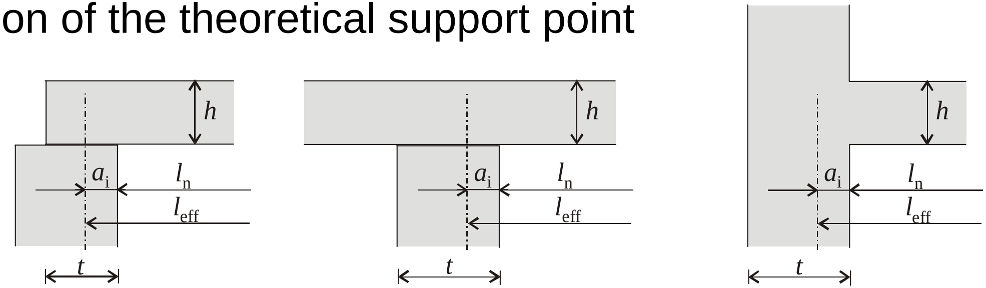
11. Approximate design of section subjected to flexure

12. Numerical example

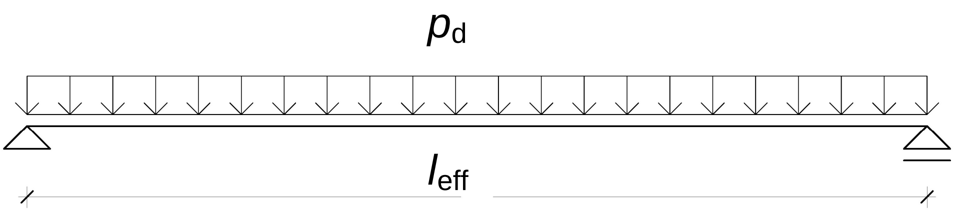
1. Modelling of an rc. beam, elements of the reinforcement system



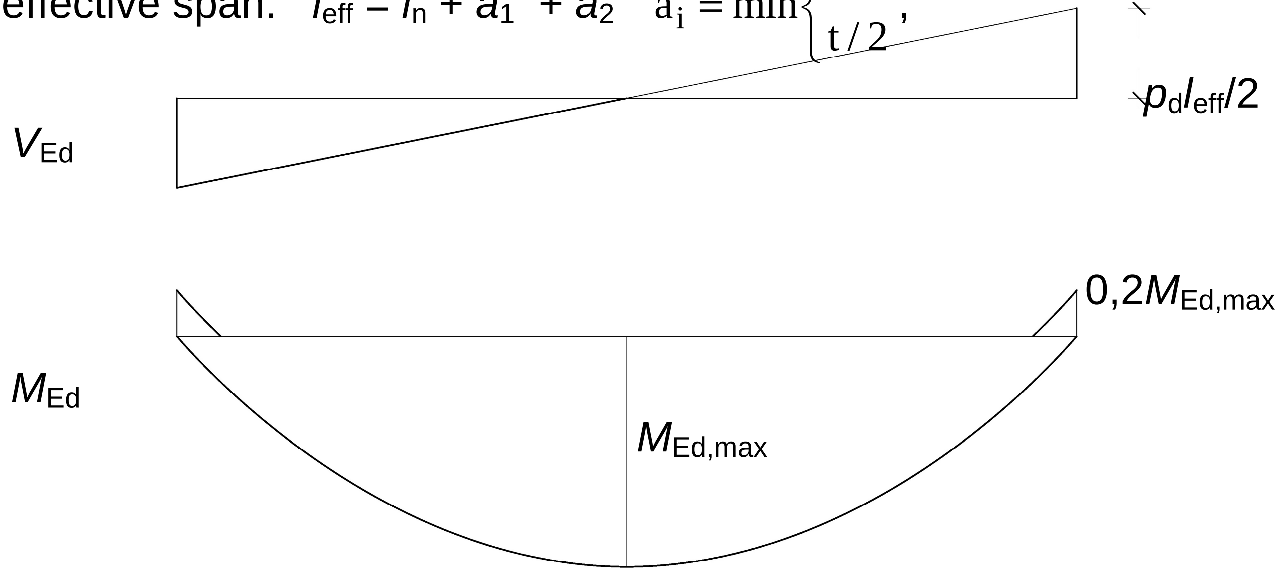
Position of the theoretical support point



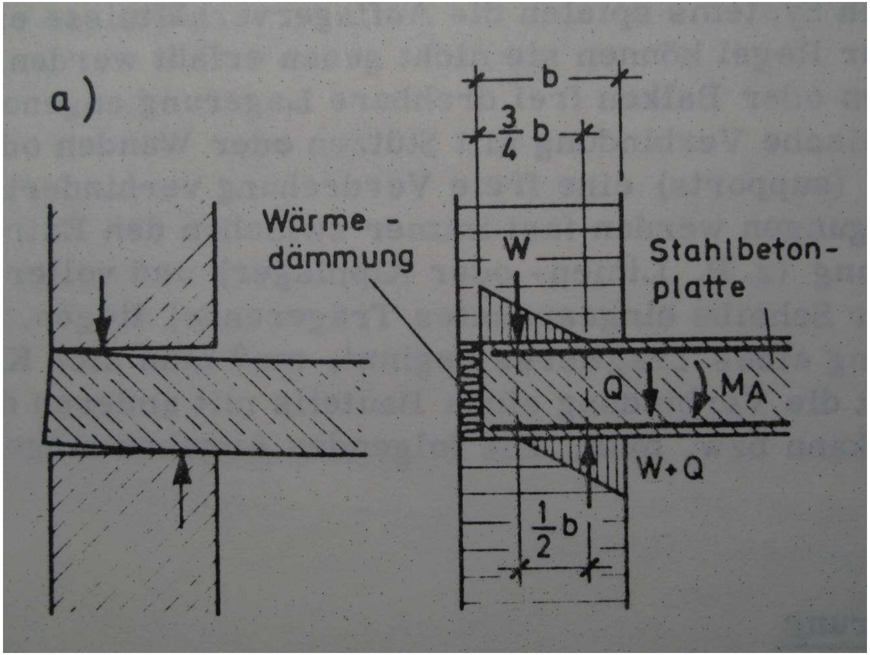
Model:



The effective span: $l_{eff} = l_n + a_1 + a_2$ $\quad a_i = \min\begin{cases} h/2 \\ t/2 \end{cases}$,



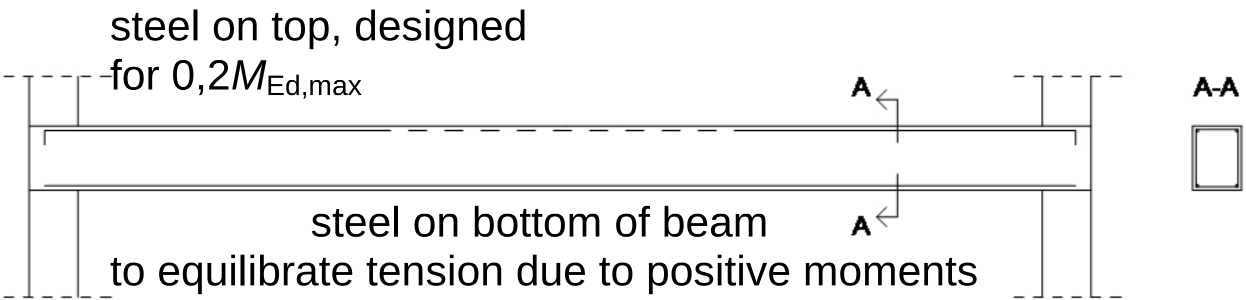
The reason of 0,2$M_{Ed,max}$ negative moment due to partial restraint at the simple supported – but by the wall above downloaded – beam end:

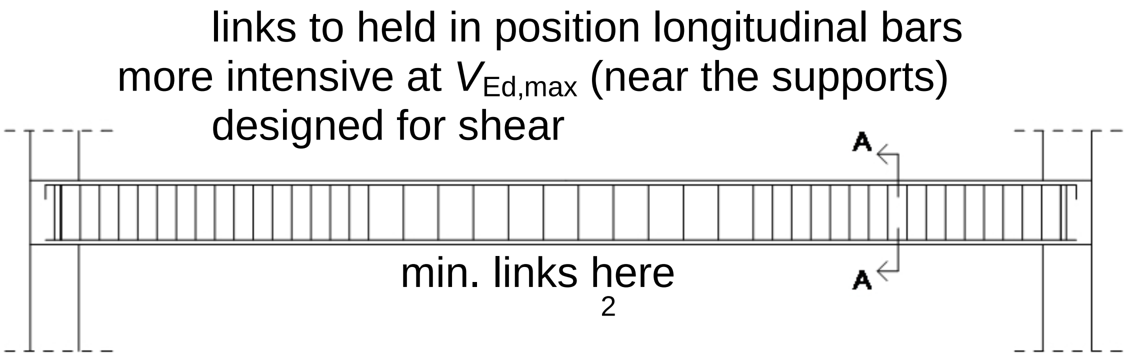


Wärmedämmung- thermal insulation

Stahlbetonplatte- rc. slab

b=t (in figure on page 3)= support length


steel on top, designed
for 0,2$M_{Ed,max}$
A
A
A-A
steel on bottom of beam
to equilibrate tension due to positive moments

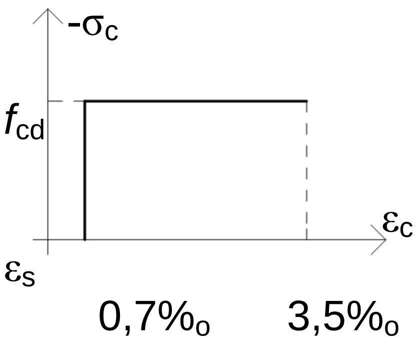

links to held in position longitudinal bars
more intensive at $V_{Ed,max}$ (near the supports)
designed for shear
A
A
min. links here

2

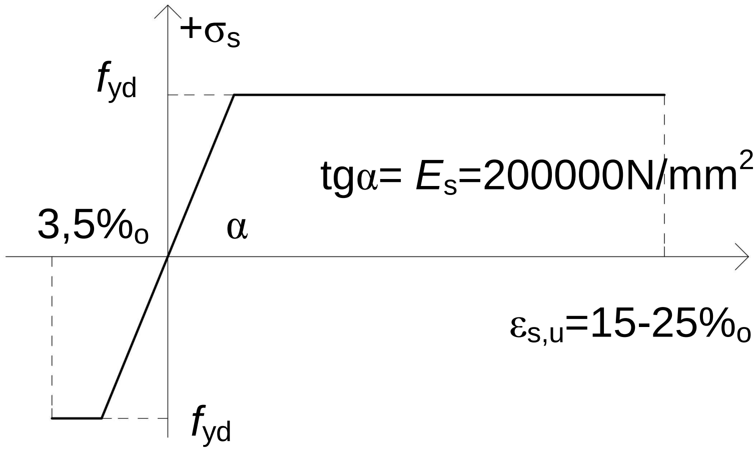
R

2. Basic suppositions of the machanical behaviour

-idealizad σ-ε diagrams of concrete and steel



Concrete



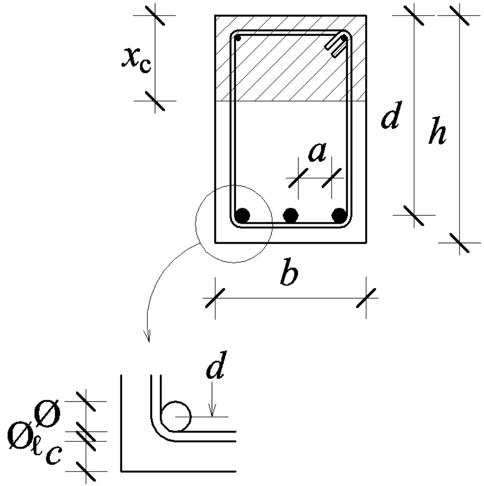
Steel

-the cross-section has vertical symmetry axis

-plane sections remain plane after deformation

-perfect bound between concrete and the surfice of the steel
(no slip of steel occurs)

3. The cross-section of an rc. beam, notations



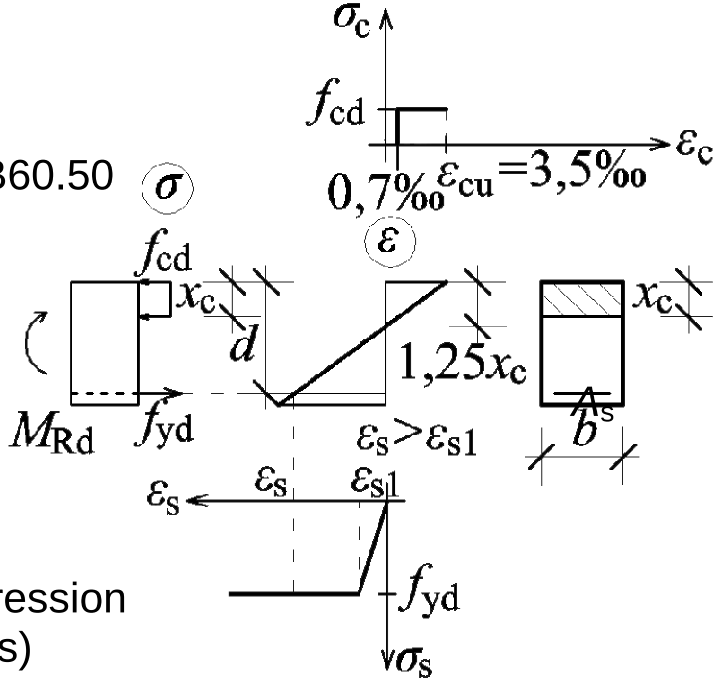
b width of the cross-section
h the height of the cross-section
c concrete cover, minimum c=20 mm
d effective depth of the section (distance between centroid of the tension steel and the extreme concrete fibre)
Ø diameter of the longitudinal bars
$Ø_l$ or $Ø_s$ diameter of links (stirrups)
a clear distance between longitudinal bars
x_c depth of the compression zone
A_s area of the tension reinforcement
A_s` area of the compression reinforcement
z internal lever arm: distance between center of the compression and tension zone respectively

4. Deformations and stresses of rectangular rc sections subjected to flexure, equilibrium conditions

Special case:

if $\varepsilon_s = \varepsilon_{s1}$ and $\varepsilon_c = 3{,}5‰$ than

$x_c = x_{co}$ $\quad \xi_o = \frac{x_{co}}{d} = 0{,}49$ for steel B60.50



Equilibrium conditions:

1): $\Sigma N = 0$

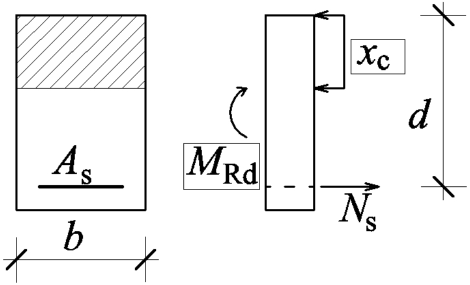
2): $\Sigma M_i = 0$

i: any point in the plane

(generally the centroid of the compression zone © or that of the tension steel (s)

5. Cases of cross-section check and design

a) Check of simply reinforced sections

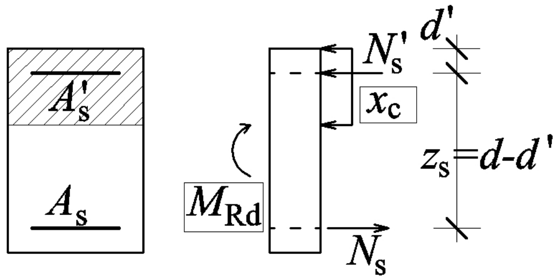


$\Sigma N = 0 \rightarrow x_c \leq x_{co}$!

$\Sigma M_c = 0 \rightarrow M_{Rd} \geq M_{Ed}$

(or: $\Sigma M_{\text{any point}} = 0 \rightarrow M_{Rd}$)

b) Check of doubly reinforced sections

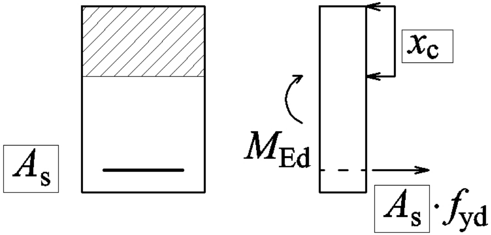


$\Sigma N = 0 \rightarrow x_c \leq x_{co}$

$\geq x'_{co} = \xi'_{co} d'$

$\Sigma M_{\text{any point}} = 0 \rightarrow M_{Rd}$

b) Design of simply reinforced sections



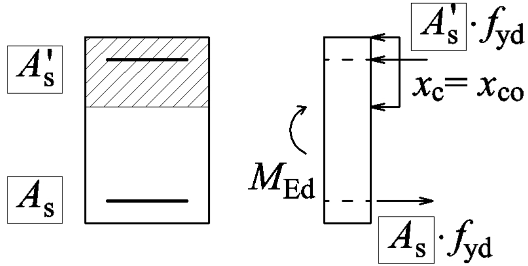
$$\underline{\Sigma M_s = 0} \rightarrow \quad M_{Ed} - b x_c f_{cd} \left(d - \frac{x_c}{2}\right) = 0$$

$$x_c = d\left(1 - \sqrt{1 - \frac{2M_{Ed}}{bd^2 f_{cd}}}\right) \le x_{co} \text{ if not} \rightarrow \ x_c = x_{co} \quad M_{Rd,o} = b x_{co} f_{cd}(d - x_{co}/2)$$

$$\underline{\Sigma M_c = 0} \rightarrow \quad A_s = \frac{M_{Ed}}{z f_{yd}} \ge A_{s,min}$$

$$z = d - x_c/2$$

d) Design of doubly reinforced section



$$\Delta M_{Rd} = M_{Ed} - M_{Rd,o}$$

$$\underline{\Sigma M_s = 0:} \quad \Delta M_{Rd} = A'_s f_{yd} z_s \rightarrow \ A'_s$$

$$\underline{\Sigma N = 0:} \quad A'_s f_{yd} + b x_{co} f_{cd} = A_s f_{yd} \rightarrow \ A_s$$

6. Special cases: steel in elastic state at rupture of the concrete

in case of $x > x_{co}$:

$$\sigma_s = \frac{560}{\xi_c} - 700$$

in case of $x < x'_{co}$:

$$\sigma'_s = \frac{560}{\xi_c} - 700$$

The equilibrium condition should be reformulated in these cases by considering the reduced stress in the steel

7. The minimum quantity of the tensile reinforcement:

Reason: the tensile reinforcement should be capable of absorbing the tensile force developing in the reinforcement after cracking of the concrete

$$A_{s,min} = \rho_{min} bd \qquad \rho_{min} = \max(1{,}5‰,\ 0{,}26\frac{f_{ctm}}{f_{yk}})$$

Tabulated in the design aids

If $A_s < A_{s,min}$, rigid failure may occur just after cracking: due to the rupture of the tension reinforcement.

8. Limitation of the compression force absorbed by the compression reinforcement

Reason: to prevent buckling of the compressed steel bars.

$$A_s \leq \rho_{max} A_c \qquad \rho_{max} = 4\% = 40‰$$

9. Principles of the realization of the necessary steel area

-$A_{\mathrm{s,prov}} \geq A_{s,requ}$ $A_{\mathrm{s,prov}} - A_{s,requ} < \varepsilon$ $\varepsilon = 0$ to cca 100 mm^2

-diameter of the bars: not too great (better crack control)
not too small (not economic, placing problem)
not too different (uniform force distribution better)

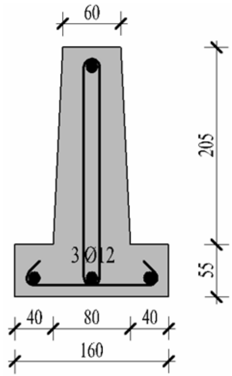
-possibility of placing of the bars in one row

-conserve symmetry of the section

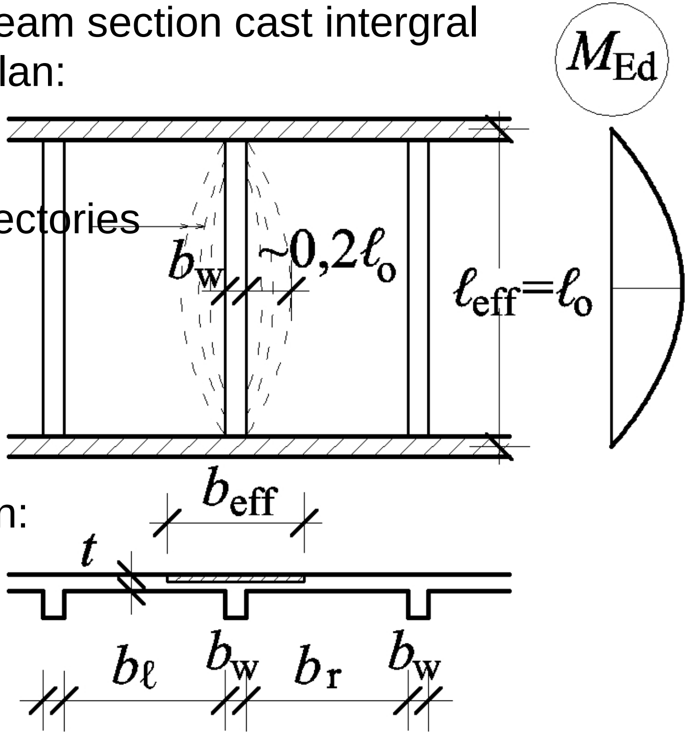
-respecting diameter of bars designed in nearby sections

10. T-sections, flanged beams

a) Prefabricated beam sections b) Flanged beams: slab and rectangular beam section cast intergral



Plan:

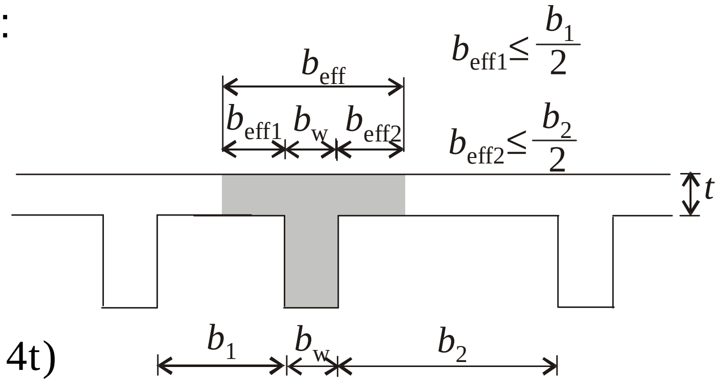


Section:

Effective width (b_{eff}) of flanged beams:

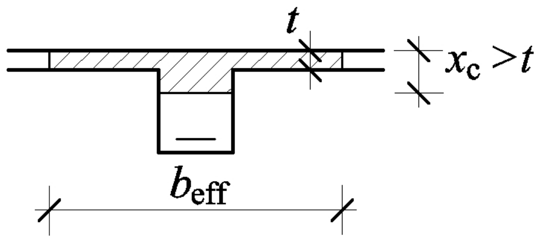
$$b_{eff} = b_w + b_{eff1} + b_{eff2} \le b$$

$$b_{effi} = \min \begin{cases} b_i / 2 \\ 0{,}2 l_o \\ 0{,}1 b_i + 0.1 l_o \\ 6t \text{ (for cantilever flange } 4t) \end{cases}$$



Reason: compression trajectories

x_c should be checked for *t* !



11. Approximate design of section subjected to flexure

To avoid the solution of 2nd order equation, for preliminary manual design calculations the following procedure can be applied:

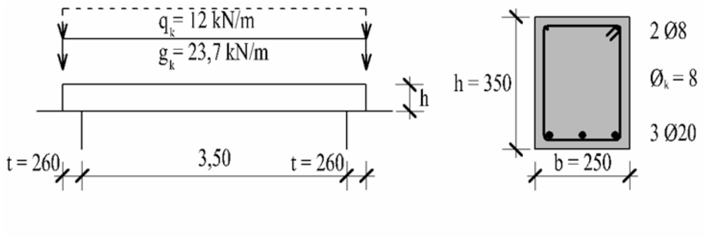
Let be $x_c = x_{co}$! $z_o = d - 0{,}5x_{co}$ $M_{Rd,o} = bx_{co}f_{cd}z_o \geq M_{Ed}$?

Variant 1: $A_s = \dfrac{M_{Ed}}{z_o f_{yd}}$ (frequently used in practice)

Variant 2: $z = d(1 - \dfrac{\xi_{co}}{2}\dfrac{M_{Ed}}{M_{Rd,o}})$ to get better approximation for the internal lever arm z

$$A_s = \frac{M_{Ed}}{zf_{yd}}$$

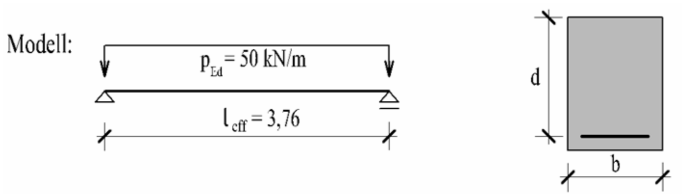
12. Numerical example



C20/25-32/KK

B 60.50

c_{nom}: 20 mm (conc. cover



$$l_{eff} = 3.5 + \min\left\{ \begin{array}{l} 2\cdot\dfrac{0,35}{2} \\ 2\cdot\dfrac{0,26}{2} \end{array} \right\} = 3.76 \text{ m}$$

Design load intensity:

$$p_{Ed} = \gamma_G\, g_k + \gamma_Q\, q_k = 1,35\cdot 23.7 + 1,5\cdot 12 = 50,0 \text{ kN/m}$$

$$M_{Ed} = p_{Ed}\frac{l^2_{eff}}{8} = 50 \cdot \frac{3{,}76^2}{8} = 88{,}36 \text{ kNm}$$

The top reinforcement will be neglected, the area of 3Ø20 on tension side:

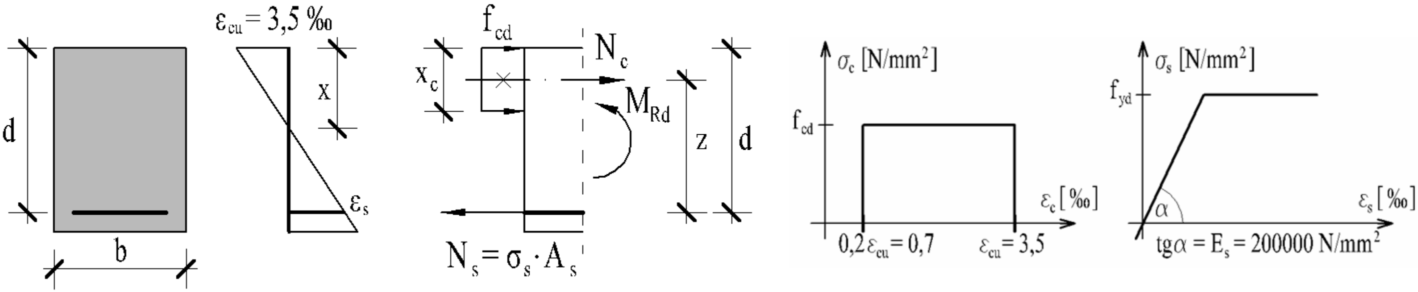
A_s = 942 mm^2 (VS. 8. o.)

Effective depth:

d = 350 - 20 - 8 - 20/2 = 312 mm

Constructional rules ($\rho_{min} = 0{,}13\%$, $A_{s,max}$= 0,04A_c) will not be checked now

Calculation of M_{Rd}



$$f_{cd} = \frac{20}{\gamma_c} = \frac{20}{1,5} = 13.3\,\text{N/mm}^2 \qquad f_{yd} = \frac{500}{\gamma_s} = \frac{500}{1,15} = 435\,\text{N/mm}^2$$

Supposing, that $\xi_c < \xi_{c0}$ (the tension steel yields at rupture of the concrete):

$\underline{\Sigma N = 0}$): $N_c = N_s$ $\qquad f_{cd} x_c b = A_s f_{yd} \Rightarrow x_c = \frac{942 \cdot 435}{250 \cdot 13,3} = 123\ \text{mm}$

Check: $\qquad \xi_c = \frac{x_c}{d} = \frac{123}{312} = 0,394 < \xi_{c0} = 0,49$ (DA. Page 7.) OK!

$(\underline{\Sigma M_c = 0})$:

$$M_{Rd} = N_s z = A_s f_{yd} z \qquad z = d - \frac{x_c}{2} = 312 - \frac{123}{2} = 251\text{mm}$$

$$M_{Rd} = 942 \cdot 435 \cdot 251 = 102,66 \cdot 10^6\ \text{Nmm} = 102,66\ \text{kNm} > M_{Ed} = 88,36\ \text{kNm}$$

OK! The section is safe!