Lecture no. 10:

**FLAT SLABS**
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1. Introduction, definitions

Flat slabs are two-way slabs supported by point-like supports.

Linear supports:
- a) column-beam support
- b) column-wide beam (dropped slab strip) support

Point-like supports of flat slabs:
- c) column (without column head)
- d) column with fungiform column head

In flat slabs the total load intensity should be carried in both perpendicular directions. Beside this, due to shear problems at column heads, the thickness of flat slab is greater:
Definition of column strips (A) and middle strips (B)

The width of *column strips* is \( \min(0.5\ell_x, 0.5\ell_y) \) in both directions:
The strips between column strips are *middle strips*
\[ m_{x}^{+O} = \frac{0.55 \, M_{x}^{+}}{l_{y}^{o}} \]

\[ m_{x}^{-O} = \frac{0.75 \, M_{x}^{-}}{l_{y}^{o}} \]

\[ m_{x}^{+l} = \frac{0.45 \, M_{x}^{+}}{l_{y}^{l}} \]

\[ m_{x}^{-l} = \frac{0.25 \, M_{x}^{-}}{l_{y}^{l}} \]

\[ l_{y}/4, \ l_{y}/4, \ l_{x} = l_{x} - l_{y}/2, \ l_{x}^{o} = l_{y}/2 \]
2. The effective span

The dashed part of the column head is regarded infinitely rigid. The peak value of the moment determined at the column axis is rounded down along the infinitely rigid part of the support:

\[ l_{x,\text{eff}} = l_x - 2 \cdot \frac{2}{3} c_x \]
\[ l_{y,\text{eff}} = l_y - 2 \cdot \frac{2}{3} c_y \]
3. Methods of analysis

**Numerical methods**
Using numerical computerized methods
(for example FEM – the finite element method)
- any ground plan layout,
- any support arrangement and
- any load distribution can be analysed with ease

**Manual methods**
As manual calculation method the *method of substitutive frames or substitutive beams* can be applied. In the latter case the moment distribution method (the Cross method) can be applied.
4. The method of substitutive beams

Principal steps of the analysis:
- determination of the moment distribution for 1 m wide strip of the slab in x and y direction respectively for the different loading schemes or for the substitutive loading, when plastic method can be applied
- multiplication of the moments by the span perpendicular to the direction of the strip
- weighing of the moments by multiplication with the weighing factors below:

<table>
<thead>
<tr>
<th></th>
<th>+M</th>
<th>-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td>55%</td>
<td>75%</td>
</tr>
<tr>
<td>Middle strip</td>
<td>45%</td>
<td>25%</td>
</tr>
</tbody>
</table>

- division of the moments by the strip width, to get the design moment for 1 m wide strip of the slab
\[ M_x^- = \frac{p_d l_x l_y^2}{11.6} \]

\[ M_x^+ = \frac{p_d l_x l_y^2}{23.2} \]
5. Moment steel distribution above columns

Due to intensive variation of moments in column strips above the columns, 67% of steel designed for the column strip should be placed in the central half width of the column strip:

Rem.: on the figure below the index $o$ stands for *column*
6. System of welded mesh reinforcement

Full lines: bottom reinforcement  Broken lines: top reinforcement

Extent of mesh overlap see at constructional rules of the design aids
7. Perimeter of flat slabs

For to create better shear transfer conditions between slab and column, the perimeter of flat slabs should be designed with cantilever or edge beam:

\[ \ell_c \leq 0.3 \ell_x \]
8. Loading schemes

To determine extreme values of positive and negative moments and shear forces at columns, different loading schemes are applied. Creation principles of loading schemes are similar to those used for continuous beams.

For $M_{\text{max}}$ above column B and $B_{\text{max}}$:

For maximum and minimum moments in the spans the chess-board rule is used:
9. Deflections of flat slabs

Deflections along column strips and middle strips in the perpendicular direction should be added:

Distance between inflection points of the deflected slab along the diagonals

\[ L = \max(\frac{L_{\text{eff},x}}{K}, \frac{L_{\text{eff},y}}{K}) \]

where \( K = 1, 2 \)
10. Punching, unbuttoning and perforation

Shear stress distributions in the slab around the column:

Punching due to $N$  unbuttoning due to $M$  perforation due to $N + M$
11. Punching of flat slabs – design for shear at the column heads

1. Danger of compression failure around perimeter $u_0$: $\nu_{Ed} \leq \nu_{Rd,max}$

where: $\nu_{Ed} = \frac{\beta V_{Ed,o}}{u_0 d}$ and $\nu_{Rd,max} = 0.5 \nu f_{cd}$

In the above expressions $d = \frac{d_x + d_y}{2}$

$\nu = 0.6 \left(1 - \frac{f_{ck}}{250}\right)$, $\Theta = \arctan \left(\frac{1}{2}\right) = 26.6^\circ$
Through the parameter $\beta$ the effect of unwanted eccentricity of the support reaction force $V_{Ed,o}$ is approximately taken into consideration:
2. No shear reinforcement is needed, if the specific shear capacity of the concrete along perimeter $u_1$ – at distance $2d$ from the column perimeter - is greater than the actual shear stress:

$$v_{Ed} \leq v_{Rd,c}$$

where:

$$v_{Ed} = \frac{\beta V_{Ed}}{u_1 d}$$

and

$$v_{Rd,c} = c f_{ct,d}$$

the parameter $c$ is tabulated in the design aids

\[ \Theta = \arctan \left( \frac{1}{2} \right) = 26,6^\circ \]
If the concrete’s shear capacity is smaller than the actual shear force:

3. **Design of the shear reinforcement**

\[ v_{Ed} \leq v_{Rd,cs} \]

\[ v_{Rd,cs} \] is the design value of the specific punching shear capacity of the slab designed with shear reinforcement along the section investigated (N/mm²):

\[ v_{Rd,cs} = 0,75v_{Rd,c} + nA_{sw}f_{ywd,ef} \sin \alpha \frac{1}{u_1d} \]

where: \[ n = \frac{1,5d}{s_r} \]  
\[ A_{sw} \] is the cross-section of the shear reinforcement along one of the perimeters investigated  
\[ s_r \] radial spacing of elements of the shear reinforcement
\[ f_{ywd,ef} = 250 + 0,25d \leq f_{ywd} \text{ (N/mm}^2\text{)} \] design value of the tensile strength of punching shear reinforcement

* \( \alpha \) is the angle between axis of the shear reinforcement and the middle plane of the slab.
4. Check of the shear capacity of the concrete along the perimeter $u_{out}$

$$v_{Ed} \leq v_{Rd,c}$$

where: $v_{Ed} = \frac{\beta V_{Ed}}{u_{out}d}$ or $v_{Ed} = \frac{\beta V_{Ed}}{u_{out,ef}d}$
12. Different types of shear reinforcement used at column heads

links cages

bent-up bars
spiral links  links cage  column longitudinal bar  welded bent ladder

welded steel dowels with jumped heads  hung link
12. Constructional rules

In transverse direction (perpendicular to the span) $s_{\text{max}} = 1.5d$ further than $2d$ from the column perimeter: $s_{\text{max}} = 2d$
Placing of links:

\[ s \leq 0.75d \]
\[ s \leq 0.3d \]
\[ s \leq 1.5d \]

Placing of bent-up bars:

\[ \leq 0.25d \]
\[ \leq 0.5d \]
\[ \geq 2d \]