Budapest University of Technology and Economics

Department of Mechanics and Structures
English courses
Reinforced Concrete Structures
Code: BMETKEPB603

Lecture no. 5:

DEFORMATIONS AND CRACKING OF R.C. STRUCTURES

Content: I. Deformations

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2. Deflection limits
3. Loads to be considered when checking deformations
4. Deflection and flaxural rigidity
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II. Cracking

1. Reasons of cracking
2. Crack direction, characteristic crack patterns
3. Limits of the crack width
4. Restoration of cracked rc structures
5. Determination of the crack width
6. Effect of some parameters on the crack width
7. Simplified check of the crack width

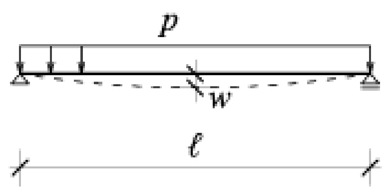
1. Why deformations should be limited?

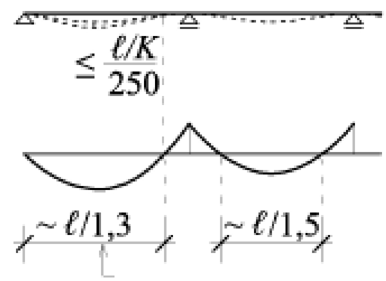
- -aesthetical reasons
- -psychological reasons
- -safety of joining constructions (partition walls, windows, tiles of the pavement)
- -functionality (canalization of rainwater)
- -modification of force distribution in arches, danger of loss of stability

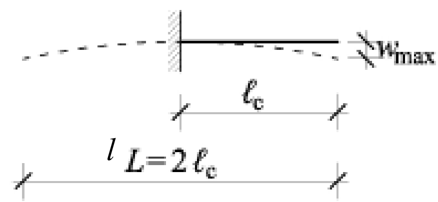
2. **Deflection limits**

$w_{max} < l\,/250$

simple sup. beams: continuous beams: cantilevers:







K tabulated in DA

here *l* is the span, *l*/K is the distance between M=0 points

3. Loads to be considered when checking deformations

Quasi-permanent load intensity:

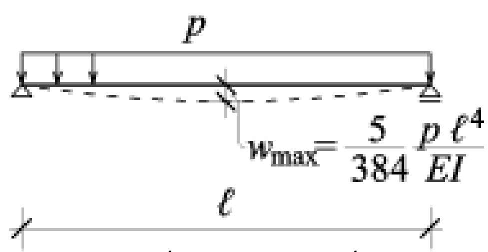
characteristic value of permanent load +
long term part of variable loads:

$$p_{qp} = g_k + \psi_2 q_k$$

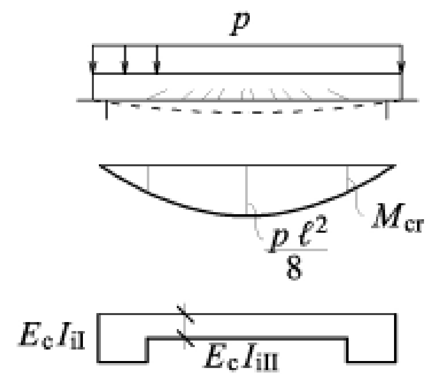
values of ψ_2 see DA table in section 4

4. Deflection and flexural rigidity

beam made of elastic material:



r.c. beam:



$$w_{max} = \frac{5}{384}\frac{pl^4}{EI} = \frac{1}{76,8}\frac{pl^4}{EI} \approx \frac{1}{80}\frac{pl^4}{EI} = \frac{Ml^2}{10EI} = \frac{1}{R}\frac{l^2}{10}$$

Effect of creep and cracking should be considered in flexural rigidity EI:

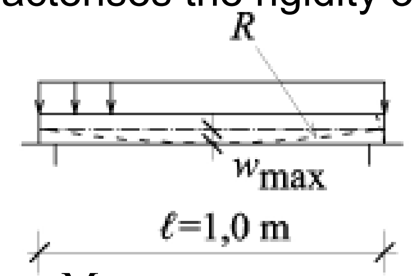
$$E = E_{c,eff} = \frac{E_{cm}}{1+\varphi_{cr}} \qquad I = I_{iII} = \eta\frac{bd^3}{12}$$

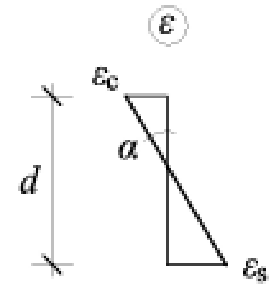
η tabulated in DA for diff. compression and tension steel %-s

5. Limitation of the slenderness ratio *l/d*

The rate L/d characterises the rigidity of r.c. members

Example:





Curvature: $y'' = \frac{1}{R} \cong \frac{M}{EI} = tg\alpha = \frac{\varepsilon_c + \varepsilon_s}{d}$

Deflection: $w_{max} \cong \frac{M\ell^2}{10EI} = \frac{1}{R}\frac{\ell^2}{10} \longrightarrow \frac{1}{R} = \frac{10 w_{max}}{\ell^2}$

Approximate curvature of an rc beam under service conditions:

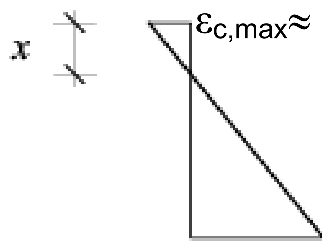
$$\varepsilon_c + \varepsilon_s \cong 0{,}85 \cdot (2{,}0\%_o + 2{,}17\%_o) = 3{,}54\%_o$$

$$\frac{1}{R} = \frac{0{,}00354}{d} = \frac{10 w_{max}}{\ell^2} \longrightarrow w_{max} = \frac{\ell^2}{2825d} \le \frac{\ell}{250} \longrightarrow \frac{\ell}{d} \le 11{,}3$$

Consequently: if ℓ/d=12 then – by considering deformations characteristic for the service state – the deflection will approximately be equal to ℓ/250

For slightly reinforced members (for example slabs): $\xi_c = \frac{x_c}{d} \cong 0{,}08$

$\varepsilon_{c,max} \approx$ 0,08x1,85=0,15 ‰



$$\varepsilon_c + \varepsilon_s = 2{,}00‰$$

$$\frac{0{,}002}{d} = \frac{10 w_{max}}{\ell^2}$$

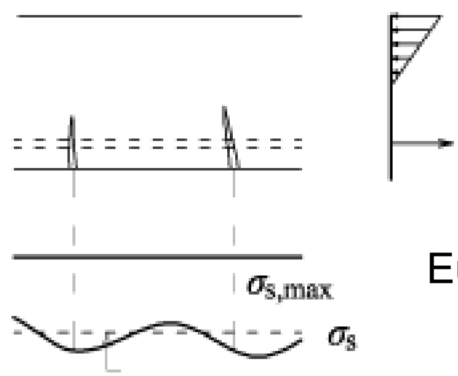
$\varepsilon_s \approx$ 0,85x2,17‰=1,85‰

$$w_{max} = \frac{\ell^2}{5000d} \le \frac{\ell}{250} \rightarrow \frac{\ell}{d} \le 20$$

Deflection problems of slabs amerge above slenderness ratios $\frac{\ell}{d} \succ 20$

$(\frac{\ell}{d})_{allowable}$ ratios on basis of more exact calculations see in DA tables!

6. The favourable effect of uncracked concrete between cracks



approximation

cont. line: real behaviour
dashed. line: approximate behaviour
difference:help ofncracked concrete between cracks

Eurocode 2:

$$w = (1-\zeta)w_1 + \zeta w_2$$

$$\zeta = 1 - 0{,}5\left(\frac{M_{cr}}{M}\right)^2 \geq 0$$

here indices 1 and 2 stand for stress states 1 (uncracked) and 2(cracked)

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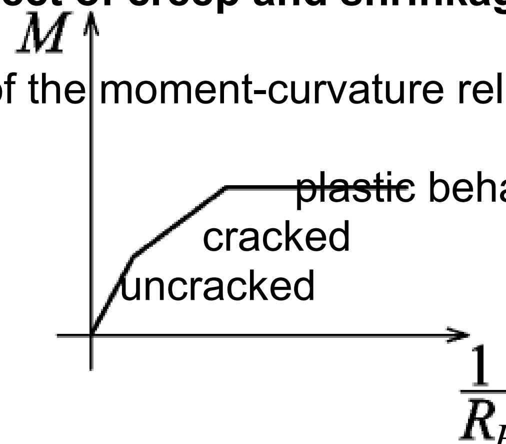
7. Effect of creep and shrinkage

Approximate character of the moment-curvature relationship of rc setions:



Curvature:

$$\frac{1}{R_{iM}} = \frac{M}{E_{c,eff} I_i}$$

Effect of creep: $E_{c,eff} = \dfrac{E_{cm}}{1+\varphi_{cr}}$ $\qquad \varphi_{cr} = \dfrac{\varepsilon_{c,pl}}{\varepsilon_{c,el}}$

Effect of shrinkage: $\dfrac{1}{R_{i,sh}}$

The total curvature:

$$\frac{1}{R_i} = \frac{1}{R_{i,M}} + \frac{1}{R_{i,sh}}$$

The most effective tool to *reduce creep deformation* is to inrease the quantity of compression steel (φ_{cr} decreases and I_i increases)

8. Simplified check of the deflection

$$\frac{\ell / K}{d} \leq \alpha \; (\ell / d)_{allowable}$$

Values of *K* for checking of deflections	
Simple supported beam or slab without cantilever	*K* = 1
Exterior span of continuous beam or slab	*K* = 1.3
Interior span of continuous beam or slab	*K* = 1.5
Flat slab	*K* = 1.2
Cantilever	*K* = 0.4

Basic values of the allowable slenderness ratio $(l/d)_{allowable}$ for rectangular sections											
Concrete strength grade	$\beta \frac{p_{Ed}}{b}$ [kN/m²] (by beams *b* is the width of the beam in m, by slabs *b*=1,0 m)										
	300	250	200	150	100	50	25	20	15	10	5
≥C40/50	13	14	14	15	17	20	25	27	30	35	47
C35/45	13	14	14	15	16	19	24	26	29	34	45
C30/37	13	13	14	15	16	19	23	25	28	33	43
C25/30		13	14	14	16	18	22	24	27	31	41
C20/25			14	14	15	18	21	23	25	29	39
C16/20				14	15	17	21	22	24	28	37
	——„beam” ——→					←—— „slab” ——					

Modification factors:

$$\alpha = \sqrt{\frac{1}{2}\beta\frac{p_{Ed}}{p_{qp}}} \qquad \beta = \frac{M_{Rd}}{M_{Ed}}\frac{500}{f_{yk}} \cong \frac{A_{s,prov}}{A_{s,requ}}\frac{500}{f_{yk}}$$

For T-sections and flanged beams another table must be used

II. Cracking

1. Reasons of cracking

$f_{ct,d} \approx 0,1\ f_{cd}$ $\sigma_{c,max} \geq f_{ct,d}$

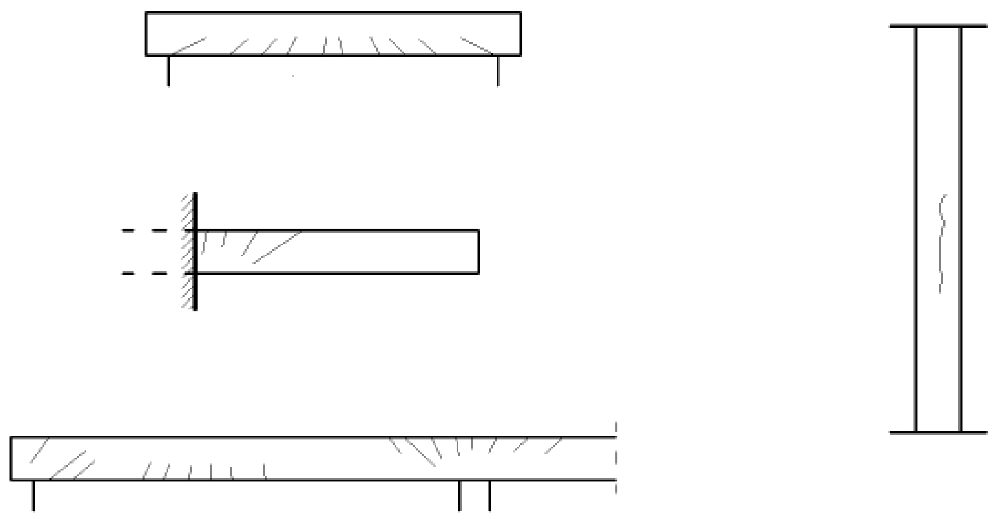
-impeded deformation (for example: shrinkage)

-temperature effects (examples: external corridor cantilever slab, fence plinth)

-tension provoced by internal forces (axial or eccentric tension, flexure, shear, torsion)

R.c. structures in service conditions are generally cracked, even water containers can be cracked.
Dilatation joints and correct support conditions (neopren pad) inhibit unwanted cracking.

2. Crack direction, characteristic crack patterns



Crack direction shows the direction of principal stresses. Diagonal cracks are called shear cracks. The most dangerous crack is is that of the column.

3. Limits of the crack width

Problems caused by cracking under quasi-permanent loads and the corresponding limits of the crack width

Aesthetical problems	0,4 mm
Corrosion in ambient variably dry and wet (XC2….XC4) or by exposure to chlorides (XD1….XD3)	0,3 mm

-When *prestressing steel* is used – due to its high sensitivity to corrosion – more strict requirements apply

-Requirement of *watertightness* is fulfilled if in serviceability state the height of the compression zone reaches 50 mm

in wet ambient	0,2 mm
in agressive ambient, in soil	0,1 mm

4.Restoration of cracked rc structures

Possible *ways of protection* against cracking:

-exclusion of factors impeding shortening caused by cinematic effects

-application of watertight flooring, over-spanning cracks

Ways of *restoration:*

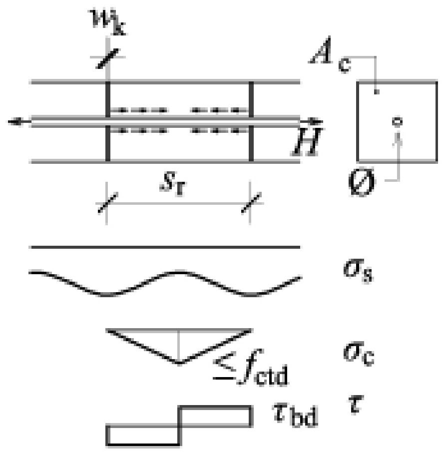
$w_{cr}>1$ mm: injection of cement milk

$w_{cr}<1$ mm: injection of epoxy resin

sealing with fibrous foil, cladding

5. Determination of the crack width

distance between cracks:



$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}),$$

$$s_{r,max} = 3{,}4c + 0{,}425\ k_1\ k_2\ \phi_s\ A_{c,eff} / A_s$$

$$\varepsilon_{sm} = \frac{\sigma_s - 0{,}4 f_{ctm} A_{c,eff} / A_s}{E_s}$$

$$\varepsilon_{cm} = 0{,}4 f_{ctm} / E_{cm}$$

c concrete cover

k_1 = 0,8 for deformed barsl, 1,6 for smooth bar surfice

k_2 = 0,5 for flexure, 1,0 for axial tension

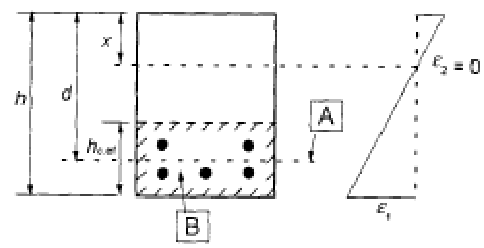
Φ_s = diameter of tension reinforcement
$A_{c,eff}$ for flexure see figures below
$A_s = A_{s,prov}$

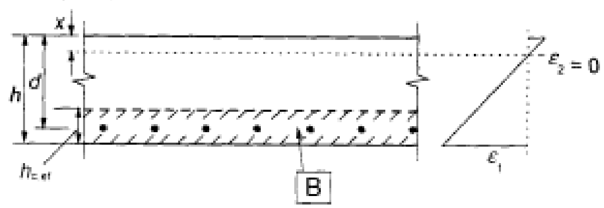
A = level of the center of A_s

B = $A_{c,eff}$,

$x = x_{II}$,

$$h_{c,ef} = \min \begin{cases} 2{,}5(h-d) \\ (h-x)/3 \\ h/2 \end{cases}$$

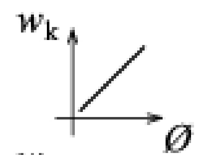




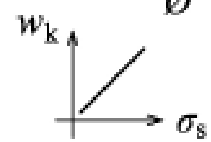
6.Effect of some parameters on the crack width

Variation of the crack width with

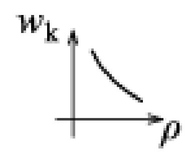
-diameter of tension steel:



-steel stress:



-steel ratio



Reduce ba diameter and increase steel ratio to control crack width!

7. Simplified check of the crack width

Determine:

$$\sigma_s \approx f_{yd} \cdot \frac{p_{qp}}{p_{Ed}} \cdot \frac{A_{s,requ}}{A_{s,prov}}$$

$$\rho = \frac{A_s}{bd} (\%)$$

and find find the greatest allowable tension steel diameter for the given limit of the crack width in the table below

When using different bar diameters:

$$\phi_{equ} = \frac{\Sigma\phi^2}{\Sigma\phi}.$$

Steel stress σ_s (N/mm^2)	Maximum steel bar diameter ϕ_{max} (mm) in function of the steel ratio and the steel stress. To satisfy the crack with limitation condition $w_k \leq w_{k,allow}$, if $c_{min,dur} \leq 20$ mm.																	
	$w_{k,allow}$ = 0,4 mm						$w_{k,allow}$ = 0,3 mm						$w_{k,allow}$ = 0,2 mm					
	Steel ratio ($\rho = A_s/bd$, %)						Steel ratio ($\rho = A_s/bd$, %)						Steel ratio ($\rho = A_s/bd$, %)					
	0,15	0,2	0,5	1,0	1,5	2,0	0,15	0,2	0,5	1,0	1,5	2,0	0,15	0,2	0,5	1,0	1,5	2,0
160	16	21	40	40	40	40	12	16	34	40	40	40	7	10	23	30	35	38
200	13	17	34	40	40	40	9	12	26	34	39	40	5	7	16	21	26	30
240	10	14	26	36	40	40	7	10	19	27	33	37	-	6	10	14	18	21
280	9	11	21	31	37	40	6	8	14	21	27	31	-	4	7	10	12	14
320	7	10	17	25	32	36	-	7	11	16	21	26	-	-	4	6	8	9
360	6	8	14	21	28	32	-	6	8	13	17	20	-	-	-	-	4	4