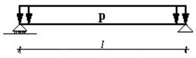
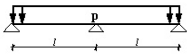
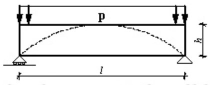
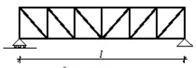
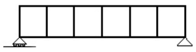
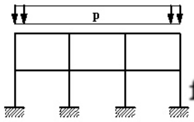
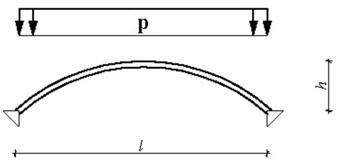
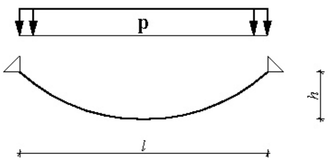
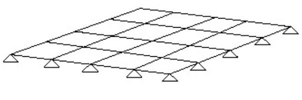
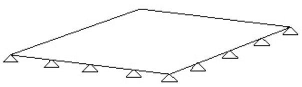
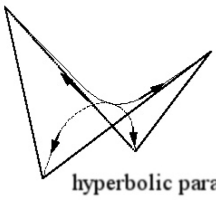
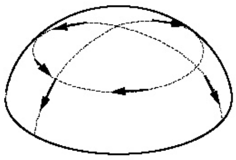
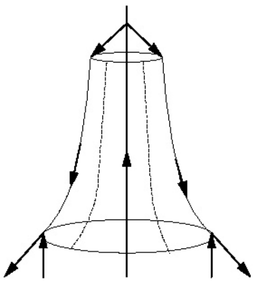
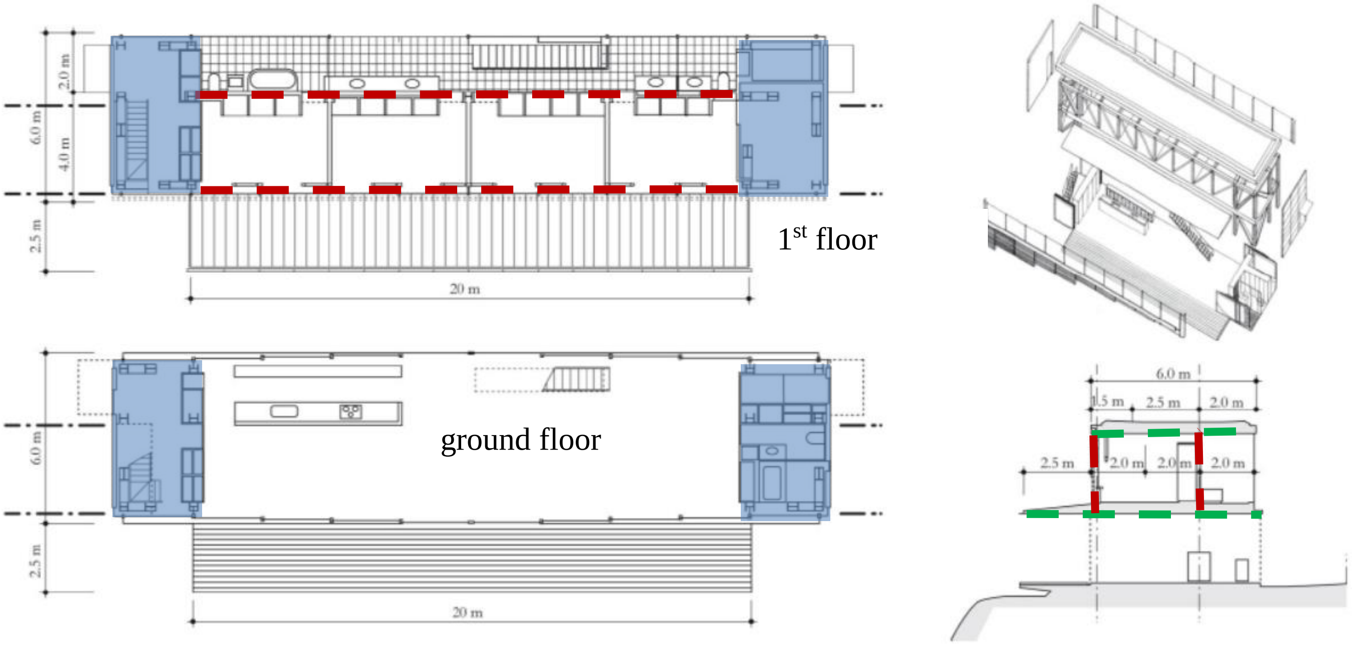
T1. LARGE SPAN STRUCTURES

A classification of the structures

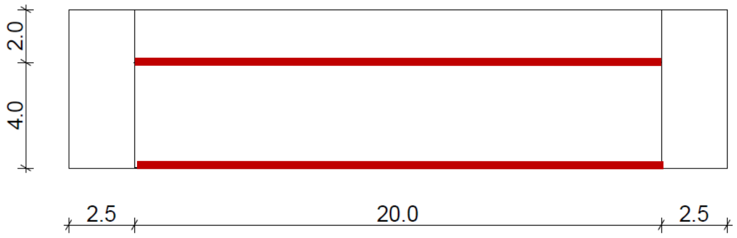
	Bended structures	Non-bended structures	
		Compression	Tension
2D	   simply supported wall beam  truss  Vierendeel truss  frame	 arch (e.g. vault)	 cable
3D	 beam grid  plate	shells  hyperbolic paraboloid  elliptic shell, dome	 membrane structure

How can we construct (approximately) bending free structures? Answer: the shape of the structure should be chosen wisely regarding the design loads.

Storey height structures



Drawings of the Picture Window House

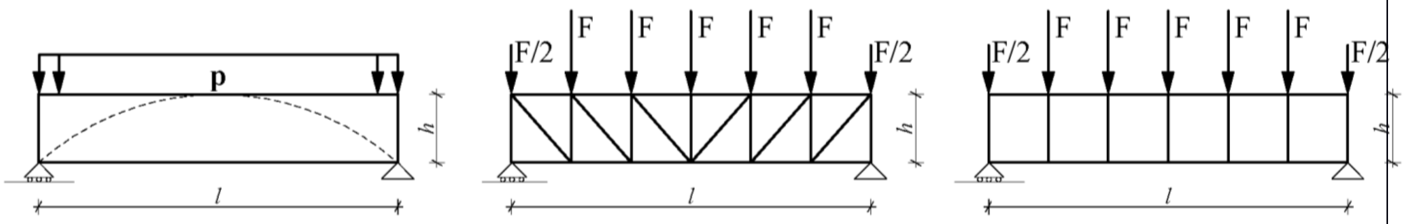


Schematic floorplan of the 1st floor

We design the floor height structural elements (marked by red lines) of the Picture Window House with three different structural solutions:

1. Wall beam
2. Truss
3. Vierendeel truss

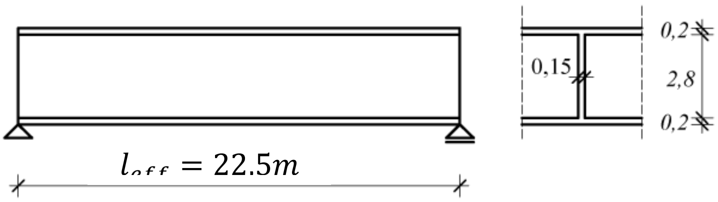
These elements hold the loads of two slabs (marked by green) and they are supported by the structures on the sides (marked by blue). The statical model of the different solutions:



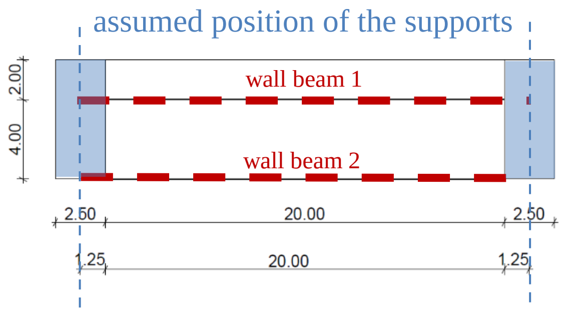
wall beam truss Vierendeel truss

Exercise 1. Wall beam

Design the steel reinforcement of the wall beam with an approximate method!

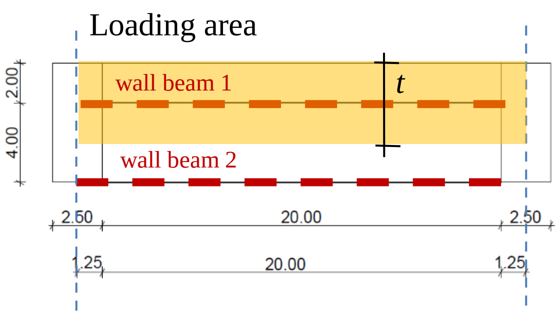


Effective length:
$l_{eff} = 20 + 2 \cdot 2{,}5/2 = 22{,}5$ m
(span + half the support width at both sides)



$l/h = 22{,}5/3{,}0 = 7{,}5$ → wall beam (the height of a 'normal' beam is usually much smaller)
Conclusion: the Bernoulli-Navier hypothesis is not valid anymore, therefore the formulas for beams cannot be used to calculate the structure. We use an approximate method instead.

Loading area:
$t = 2 \cdot 2{,}0 = 4{,}0$ m (for 'wall beam 1')
Now we calculate only the highest loaded element: 'wall beam 1'. (For 'wall beam 2', the loading area is only $t = 2{,}0$ m)



Material data:
concrete: $f_{cd} = 16{,}6$ N/mm^2 (C25/30)
steel: $f_{yd} = 435$ N/mm^2 (B500)
density of reinforced concrete (walls, slabs):

$$\rho = 25\,kN/m^3$$

Loads: ('*k*' subscript denotes a characteristic value)
live loads + loads of the partitioning walls:

$$q_k = 3\,kN/m^2$$

dead load of a slab = layers + self weight of the plate:

$$g_k^{layers} = 2{,}0\,kN/m^2$$

$$g_k^{plate} = 0{,}2 \cdot 25 = 5\,kN/m^2$$

$$g_k^{slab} = g_k^{layers} + g_k^{plate} = 7\,kN/m^2$$

dead load of the wall beam = dead load of the beam + plaster:

we assume $t = 9$ cm of plaster with $\rho^{plaster} = 18\,kN/m^3$ density

$$g_k^{wall} = h^{wall}(t^{wall}\rho^{wall} + t^{plaster}\rho^{plaster}) = 2{,}8 \cdot (0{,}15 \cdot 25 + 0{,}09 \cdot 18) = 12\,kN/m$$

Model:

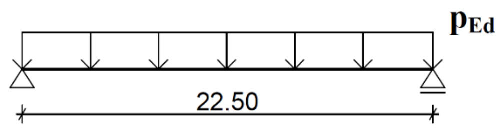
The model of the wall beam is a simply supported beam loaded by uniformly distributed load. We need to transform the loads to distributed loads over a line and then calculate the total load. Furthermore, the values must be multiplied by the safety factors.

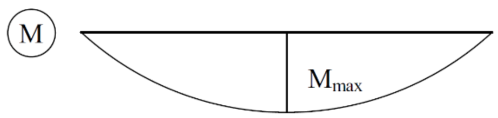
safety factor for self-loads: $\gamma_G = 1{,}35$

safety factor for live loads: $\gamma_Q = 1{,}50$

The total load of 'wall beam 1' = loads coming from the self-load of 2 slabs transformed to a line + self-load of the wall beam + live loads of 2 slabs transformed to a line. Each characteristic load has to be multiplied by the appropriate safety factor! The subscript 'd' denotes design values, 'e' denotes effect.

$$p_{Ed} = \gamma_G \cdot \left(2 \cdot g_k^{slab} \cdot t + g_k\right) + \gamma_Q \cdot 2 \cdot q_k \cdot t = 1{,}35 \cdot (7{,}0 \cdot 2 \cdot 4 + 12) + 1{,}5 \cdot 2 \cdot 3 \cdot 4 = 130\, kN/m$$



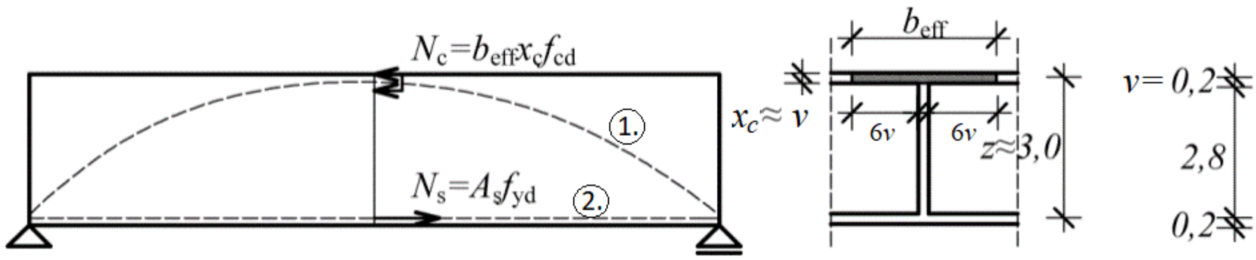


The maximal bending moment:

$$M_{Ed} = M_{max} = \frac{p_{Ed} \cdot l_{eff}^2}{8} = \frac{130 \cdot 22{,}5^2}{8} = 8230 kNm$$

The amount of the necessary steel reinforcement depends on the maximal bending moment. Concrete can bear no tension, therefore where tension would occur in the structure steel reinforcement is needed. For simply supported beams, the bottom part of the beam is under tension.

Investigation of the middle cross-section, arch analogy:



① compressed concrete zone (analogy: arch)

② steel reinforcement in tension (analogy: tie-bar)

We assume equilibrium between the resultant forces of the compressed and tensional zones: $N_c = N_s$, where

$N_c = A_c \cdot f_{cd} = b_{eff} \cdot x_c \cdot f_{cd}$, where A_c is the area of the compressed concrete zone

$N_s = A_s \cdot f_{yd}$, where A_s is the area of the steel reinforcement (unknown)

Furthermore, we assume equilibrium between the internal forces and the bending moment:

$M_{Ed} = N_s \cdot z = N_c \cdot z$

The slabs and the wall beams are assumed to work together, as a result x_c is assumed to be in the upper slab. With $x_c \approx v$ and z is the distance between the steel reinforcement (considered at the middle of the bottom slab) and the centroid of the compressed concrete zone.

$$b_{eff} = 6v + t_{wall} + 6v = 6 \cdot 0{,}2 + 0{,}15 + 6 \cdot 0{,}2 = 2{,}55m,$$

where t_{wall} is the thickness of the wall beam.
As at the middle of the simply supported wall beam the compressed zone can be assumed being in the upper slab, it lets the opportunity for the archic to open a door at the middle of the wall beam.

Designing:
From the equilibrium equations:

$$N_s = N_c = \frac{M_{Ed}}{z} = \frac{8230}{3} = 2745kN$$

$$A_s^{required} = \frac{N_s}{f_{yd}} = \frac{2745 \cdot 10^3}{435} = 6306mm^2$$

Let's choose 18 pieces of $\phi 22$ steel wires:

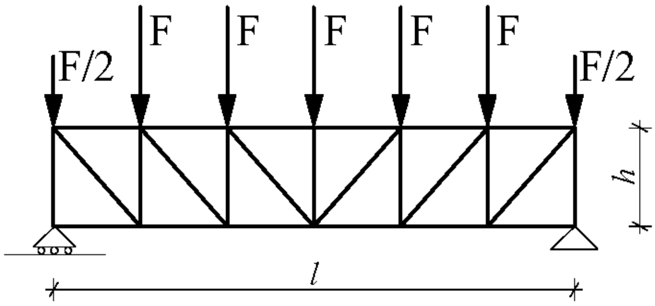
$$A_s^{provided} = 18 \cdot \frac{22^2\pi}{4} = 6842mm^2$$

Checking of the compressed zone:
We must check whether the assumed compressed zone ($A_c = b_{eff} \cdot x_c$) is large enough to bear the $N_c = 2745kN$ force. The maximal stress is f_{cd} and the area of the zone is fixed, therefore the force is limited by:

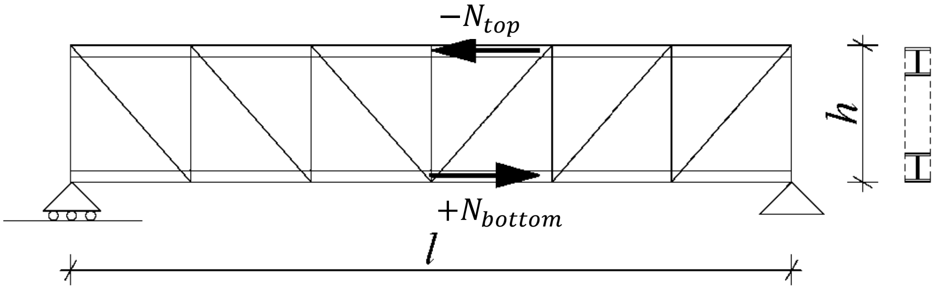
$$N_{c,max} = A_c \cdot f_{cd} = 2550 \cdot 200 \cdot 16{,}6 \cdot 10^{-3} = 8466kN.$$

Since $N_{c,max} > 2745kN$, the area of the compressed zone is large enough.

Exercise 2. Truss



Let's assume, that the loads, effective length and the height of the structure are the same as before: $p_{Ed} = 130\frac{kN}{m}, l_{eff} = 22{,}5m, h = 3m$. Instead of calculating the loads on the joints F from the distributed load, we apply a simplified approximate method to design the bottom and top chords of the truss.



The top chords are in compression, while the bottom chords are in tension. We calculate the forces in the top and bottom chords at the maximal bending moment (in the middle of the structure). The model of the truss is simply supported; therefore, the maximal bending moment is:

$$M_{Ed} = M_{max} = \frac{p_{Ed} \cdot l^2}{8} = \frac{130 \cdot 22{,}5^2}{8} = 8230kNm.$$

There are two equilibrium equations:

$$N_{top} = N_{bottom}$$
$$M_{Ed} = N_{top} \cdot h$$

We carry out the same steps as for the wall beam to determine the force:

$$N_{Ed} = N_{top} = N_{bottom} = \frac{M_{ed}}{h} = \frac{8230}{3} = 2745kN.$$

N_{Ed} is the amplitude of the largest compression force the top chords of the truss must resist. The resistance depends on the strength of the material and the slenderness of the beam. The strength must be reduced by the buckling reduction factor depending on the slenderness. In this problem, we can assume, that the reduction factor is $\chi_y = 0{,}7$. Then the necessary amount of cross section area:

$$A_{s,req} = \frac{N_{Ed}}{\chi_y \cdot f_{y,d}} = \frac{2745 \cdot 10^3}{0{,}7 \cdot 235} = 16{,}7 \cdot 10^3 mm^2$$

We choose HEA 450 beam ($A_s = 178{,}5cm^2$) from the steel cross section catalogue.