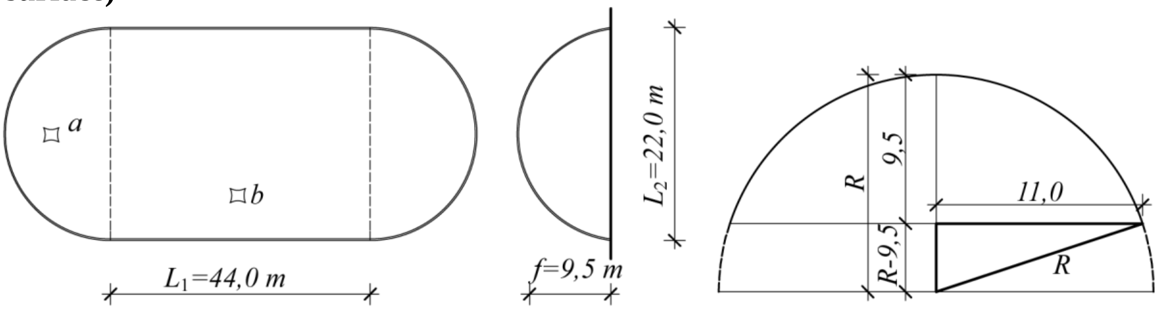
T6/1. – Verification of a pneumatical tent

A small football court is covered by a pneumatical tent for wintertime.
Exercise 1: Calculate the internal forces in the tent material at *point a* **(elliptic surface) and at** *point b* **(parabolic surface)**



<u>Geometry:</u>

$R^2 = (L_2/2)^2 + (R - f)^2$

$R^2 = 11^2 + R^2 - 19R + 9{,}5^2$

$R = 11{,}12 m$

How much additional pressure is needed so that the tent bears the occurring loads?

<u>Loads:</u>
Snow load is assumed to be equally distributed on the surface: $\mathbf{s_k=1{,}0}$ **kN/m²**
Design cases:
- normal snow load: $s_d = \gamma_Q \cdot s_k = 1{,}5 \cdot 1{,}0 = 1{,}5$ kN/m²

→ necessary pressure: $P_{nec}=P_{req}\cdot\gamma_P$ ($\gamma_{P,fav}$: safety factor of tensioning force in **favorable** case)
P_{req} = 1,5 kN/m² = 0,015 Atm *(atmosphere)*
(1 A=10⁵Pa=10⁵N/m²=100 kN/m²)

- exceptional snow load: $s_d = 2 \cdot s_k = 2 \cdot 1{,}0 = 2{,}0$ kN/m² *(two times more than the normal)*

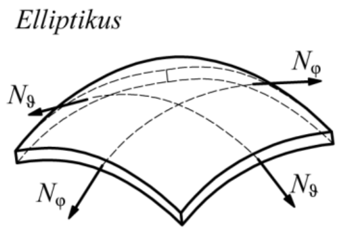
→ necessary pressure: P_{nec}=2,0 kN/m² = 0,02 Atm

To how much pressure shall we design the material of the tent?
If initial pressure and snow load are both present, in the ideal case the tent is ~stress-free. If the snow melts the external pressure ceases → The internal pressure is balanced by the stress in the tent material.

$P_{max} = P_{req} \cdot \gamma_{P,unfav} = 2{,}0 \cdot 1{,}3 = 2{,}6$ kN/m² ($\gamma_{P,unfav}$: safety factor of tensioning force in **unfavorable** case)

In normal case the internal (over)pressure is 0,015 Atm. In case of exceptional snow load the pressure shall be raised to 0,02 Atm. The material of the tent should be designed to bear 0,026 Atm pressure.

<u>Internal forces of the material of the tent at point „*a*”:</u> → elliptic surface (curved in both directions)



$p_{E,d} = \frac{N_\varphi}{R_\varphi} + \frac{N_\vartheta}{R_\vartheta}$ → static equilibrium equation. Bidirectional Barlow's formula.

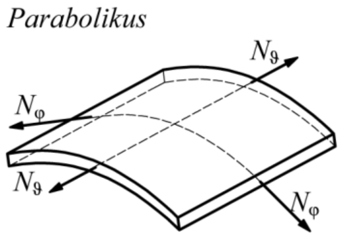
$R_\varphi = R_\vartheta = 11{,}12$ m

$N_\varphi = N_\vartheta$ → $N_\varphi = \frac{1}{2} 2{,}6 \cdot 11{,}12 = 14{,}46$ kN/m *(shear-free shell → membrane)*

Usually the strength of the tent material is given for a 5 cm × 20 cm size sample. The characteristic value of tensile strength is given.

The material safety factor is: $\gamma_{tent} = 2{,}5$.
$N_{Ed} = N_{\varphi} = 14{,}46$ kN/m → $14{,}46 \cdot 1000/20 =$ **723 N/5cm**
$\boldsymbol{N_{Rd}} \geq 2{,}5 \cdot 723 =$ **1807,5 N/5cm**! → tensile strength of an „average” tent material

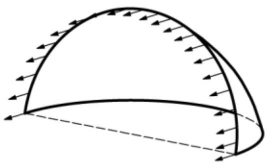
<u>Internal forces at point „*b*”:</u> → parabolic surface (curved in one direction)



$$p_{E,d} = \frac{N_{\varphi}}{R_{\varphi}} \rightarrow N_{\varphi} = p_{E,d} \cdot R_{\varphi} = 2{,}6 \cdot 11{,}21 = 28{,}9 \text{ kN/m}$$

→ $28{,}9 \cdot 1000/20 =$ **1445 N/5cm**

$\boldsymbol{N_{Rd}} \geq 2{,}5 \cdot 1445 =$ **3612,5 N/5cm**! → tent material for 4000 N/5cm tensile strength is easily available!



(the membrane force N_{ϑ} equals to the membrane force N_{ϑ} calculated at the elliptic part! see: 3D sketch)

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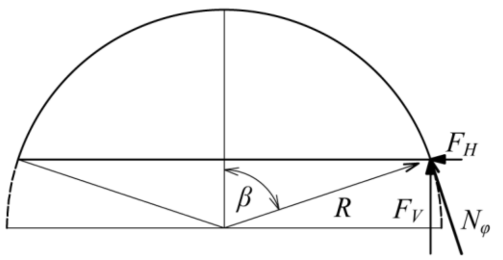
Exercise 2: Calculate the minimal dimensions of the foundation by general approximations.

a) The foundation at the spherical part:

$$\sin\beta = \frac{L_2/2}{R} = \frac{11{,}0}{11{,}12} \rightarrow \beta = 81{,}58°$$



$$F_{V,a} = N_{\varphi} \cdot \sin\beta = 14{,}46 \cdot \sin 81{,}58° = 14{,}3 \text{ kN/m}$$

$$F_{H,a} = N_{\varphi} \cdot \cos\beta = 14{,}46 \cdot \cos 81{,}58° = 2{,}1 \text{ kN/m}$$

The vertical force is balanced by the self-weight of the foundation:

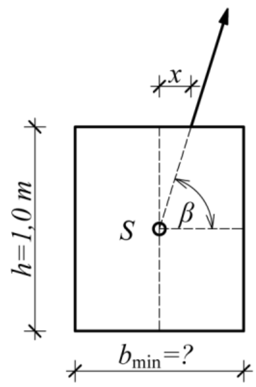
$$G_{alaptest} \geq F_{V,a} \rightarrow h \cdot b_{min} \cdot \rho_{vb} \cdot \gamma_{G,inf} \geq F_{V,a} \rightarrow b_{min} \geq \frac{14{,}3}{1{,}0 \cdot 25{,}0 \cdot 0{,}9} = 0{,}64 \rightarrow$$ let’s define $\boldsymbol{b_1}$ **= 0,7 m**

The horizontal force is balanced by the curved geometry of the foundation. The internal normal force in the circular foundation: (Barlow’s formula)

$$N_{alap} = F_{H,a} \cdot \frac{L_2}{2} = 2{,}1 \cdot 11{,}0 = 23{,}1 \text{ kN}$$ *(compression!)*

Where shall the tent be fixed in order to avoid the twisting force on the foundation?
→ The line of action of the force acting at the fixing point should meet the centroid of the foundation.

$$x_a = \frac{h}{2} \cdot \text{ctg}\, 81{,}62° = 0{,}073 \text{ m from the centerline.}$$



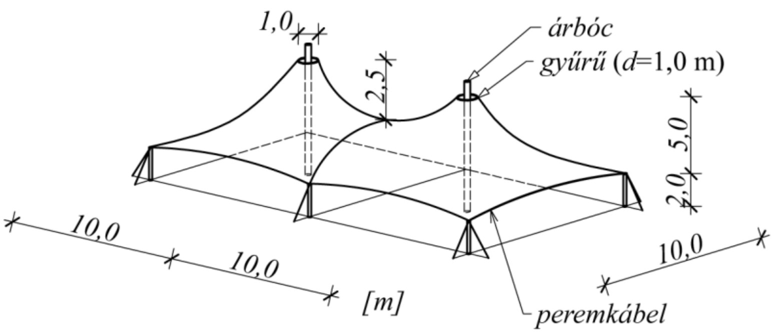
b) The foundation of the cylindrical part:

N_{φ} force is double the value of the one in the spherical part → linear correlation applies in all calculations, so: $b_{min} \geq 2 \cdot 0{,}66$ → $\boldsymbol{b_2}$ **= 1,4 m**

The position of the fixing point depends only on the tangent of the tent at the fixing point, so: $\boldsymbol{x_b} = x_a =$ **0,073 m**

The horizontal $F_{H,b}$=4,2 kN/m force is balanced by the soil (active soil pressure ?aktív földnyomás).

T6/2. – Tensioned tent structure



Pre-tensioning: ~1,0 kN/m applied at the edge cables.

Hyperbolic surface → complicated calculation based on FEM software

Approximate manual calculation (for pre-design and supervision of computer-based calculations)

1. How much internal force is caused in the mast when applying the pre-tensioning force to the tent?
As a simplification we assume that the vertical component of the pre-tensioning is ~0,5 *kN/projected meter* at the external edges *(at the edges the angle of the tent is ~30°).*

$$N_{mast}^{pre} = \frac{2\cdot(20+10)\cdot 0{,}5}{2} = 15{,}0\ \text{kN}$$

2. How much internal force is caused in the mast and in the tent in case of exceptional snow load? (s_k=1,0 kN/m^2) Is a tensile strength of $N_{tensile}$=4500 N/5 cm enough to bear the load? (γ_{tent}=2,5)

Exceptional snow load s_d=2,0·1,0=2,0 kN/m^2
Self-weight: ~ 1 kg/m^2 → always negligible in case of tents

$$N_{mast}^{snow} = \frac{20\cdot 10\cdot 2}{2} = 200\ \text{kN} \rightarrow N_{mast} = N_{mast}^{snow} + N_{mast}^{pre} = 215\ \text{kN}$$

<u>Internal force in the tent material at the top ring:</u>

Perimeter of the ring: $K = 1{,}0\cdot\pi = 3{,}14\ \text{m}$

Vertical component of the force in the tent: $N_{p,V} = \frac{215{,}0}{3{,}14} = 68{,}47\ \text{kN/m}$

Angle of the tent at the ring ~45°: $N_p = N_{p,V}/\cos 45° = 96{,}83\ \text{kN/m} \rightarrow 96{,}83\cdot\frac{1000}{20} = 4841{,}6\ \text{N/5cm}$

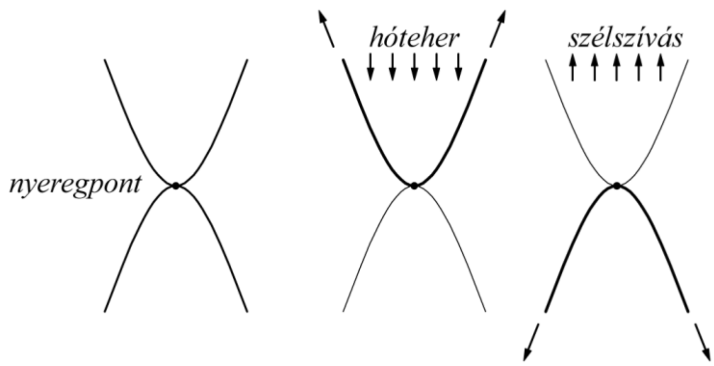
$N_{Rd} = \frac{N_{tensile}}{\gamma_{tent}} = \frac{4500}{2{,}5} = 1800\ \text{N/5cm} \rightarrow n = \frac{4841{,}6}{1800} = 2{,}69 \rightarrow n$=3 layers of tent material are needed.

At the ring the tent is made of 3 layers of material! *(Or we could increase the size of the ring.)*

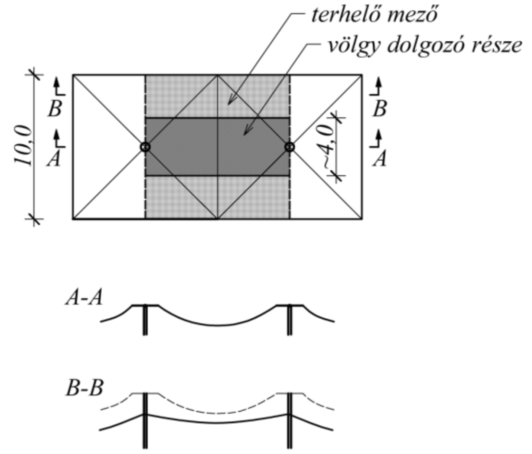
<u>Test the "valley" too. How much force is caused by the exceptional snow load?</u>
Note: hyperbolic surface:

$K_\varphi \times K_v < 0$
(Gauss curvature)

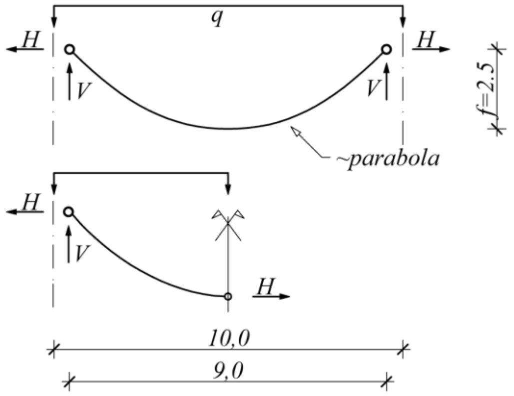
In case of opposite external force, the other direction acts. In case of tensioned tent a hyperbolic surface is always required.



Floor plan:



Static model – *parabola form is assumed*



Loading zone: $A = 10 \cdot 10{,}0$ m. Load-bearing zone: 4,0·9,0 m
(4,0 meter: as an assumption!)

The steep part bears the loads, because at the sides the curvature is too little, and these parts are not connected directly to the ring.

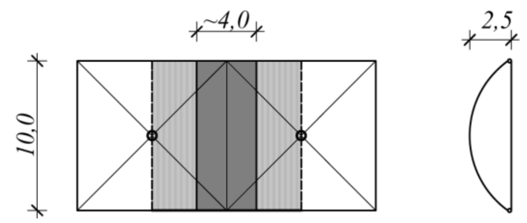
The vertical loads of the zone: $q = \frac{s_d \cdot A}{9{,}0} = \frac{2{,}0 \cdot 10 \cdot 10}{9} = 22{,}2$ kN/m at the 4 m zone!

At the saddle point the internal force in the tent based on the approximation:

$$N_{\text{tent}}^{\text{nyereg}} = H = \frac{q \cdot L^2}{8f} = \frac{22{,}2 \cdot 9{,}0^2}{8 \cdot 2{,}5} = 90\,\frac{\text{kN}}{4\text{m}} = 22{,}5\,\frac{\text{kN}}{\text{m}} = \frac{22{,}5}{20} \cdot 10^3\,\frac{\text{N}}{5\text{cm}} = 1125\,\frac{\text{N}}{5\text{cm}} < N_{Rd} \text{ OK!}$$

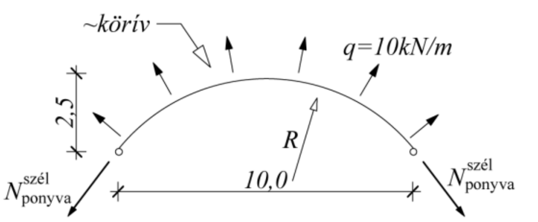
If one layer wouldn't be enough at the saddle point, then a stronger material must be chosen. In this case there are 3 layers at the ring, and the layers are gradually decreasing downwards until we reach the 1-layer thickness at the saddle point.

3. Verify the tent for wind load! w_d=1,0 kN/m^2



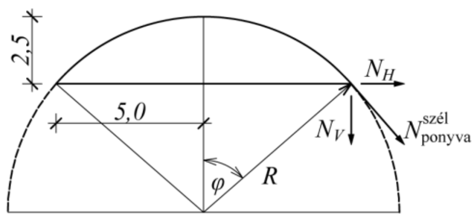
Suction: direction of the opposite curvature (to the previous one) is acting (similar to lightweight roofs)
Loading zone: $A = 10{,}0 \cdot 10{,}0$ m. Load-bearing zone: 4,0·10,0 m
(width of the load-bearing zone is estimated ...)

$q = 10\ kN/m$ (4 m wide zone)
form: assumed to be circular, because in this case we can use the Barlow's formula. *(simplification, approximation...)*



<u>Geometry:</u>
$R^2 = 5^2 + (R-2{,}5)^2$
$R = 6{,}25$ m

$$N_{\text{tent}}^{\text{wind}} = q \cdot R = 10 \cdot 6{,}25 = 62{,}5\,\frac{\text{kN}}{4\text{m}} =$$
$$= 15{,}625\,\frac{\text{kN}}{\text{m}} = \frac{15{,}625}{20} \cdot 10^3\,\frac{\text{N}}{5\text{cm}} = 781{,}25\,\frac{\text{N}}{5\text{cm}}\ (N_{\text{Ed}})$$



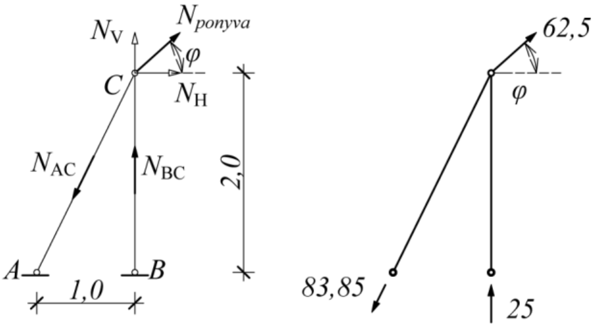
$N_{\text{Rd}} \geq N_{\text{Ed}}$, 1800,0>781,25 → One layer is OK!

$\sin\varphi = \frac{5}{6{,}25} \rightarrow \varphi = 53{,}13°$

4. Assess the supporting structure at the edges!
AC rod is tensioned, *BC* rod is compressed!

$N_{H}^{wind}=N_{tent}^{wind}\cdot\cos\varphi=62{,}5\cdot\cos 53{,}13°=37{,}5$ kN
$N_{V}^{wind}=N_{tent}^{wind}\cdot\sin\varphi=62{,}5\cdot\sin 53{,}13°=50$ kN

Equilibrium of hinge **C**:



$\sum F_H=0$ $N_{AC,H}=N_{H}^{wind}=37{,}5$ kN (←)
$N_{AC,V}=2\cdot 37{,}5=75$ kN (↓)
$N_{AC}=83{,}85$ kN parallel to the rod.

$\sum F_V=0$ $N_{BC}=50{,}0-75{,}0=-25{,}0$ kN compression in *BC* rod!

Wire rope pl.:
FUX T6x37 ϕ 22mm – Steel-core, Load bearing capacity: min. 273 kN
Required capacity: 3·83,85=253 kN, (Safety factor of 3!)

For further details about joints in tent structures please visit the ***Membrane Detail*** webpage by Bálint Füzes *(http://www.membranedetail.com/)*

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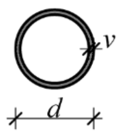
Compressed steel column
d=80 mm, v=4mm, $\chi\cong 0{,}6$
$f_{yd}=235$ N/mm^2, $A=\left(40^2-36^2\right)\cdot\pi=955$ mm^2



$N_{Rd}=\chi\cdot A\cdot f_{yd}=0{,}6\cdot 955\cdot 235\cdot 10^{-3}=134{,}6\ \text{kN}>N_{Ed}=25\ \text{kN}\ \rightarrow$ MF!
Note: Usually the supporting rods are designed to be slant, because during the adjustment of the cables, some displacement is expected. It would be embarrassing to see "almost" vertical columns.

Additional info:
Dezső a sátras programjával is végigszámolta az adott geometriát az adott terhekkel. Érdekességként megemlíthető, hogy az ottani pontos számításhoz képesti eltérés a bonyolult geometriához képest egészen jónak tekinthető. Az értékek tehát:

- *árbóc erő az előfeszítésből: 13 kN (kézi: 15 kN)*
- *árbóc erő a hóteherből: 136 kN (a szumma függőleges teher a modellen csak 330 kN a lecsúszás miatt) (kézi: 200 kN)*
- *ponyva igénybevétele a völgyben hóteherre: 18 kN/m (kézi: 22,5 kN/m)*
- *ponyva igénybevétele szélre a nyeregben: 10 kN/m (kézi: 15,6 kN/m)*