T3/1 BASIC ANALYSIS OF HYPAR SHELLS BASED ON MEMBRANE THEORY

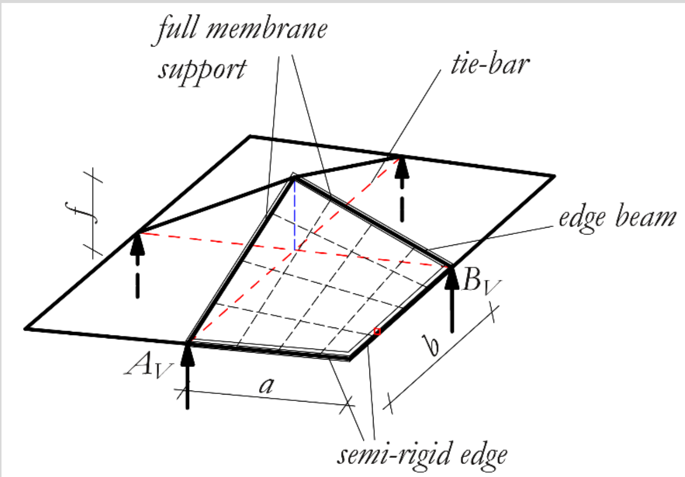
THEORY

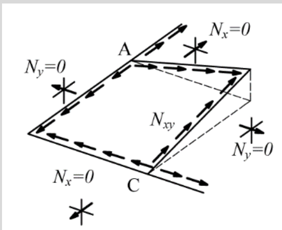
Basic analysis of hypar shells with straight boundary edges (=quadrilateral shells) is presented below, based on membrane theory of shells.
The presented method is valid as long as:

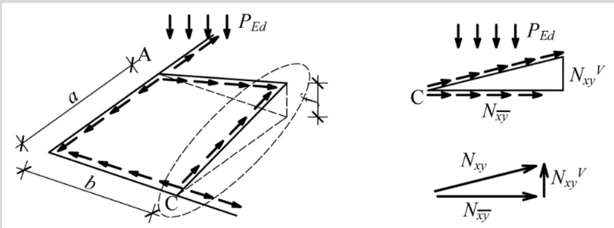
- *the VERTICAL (p_z) load is equally distributed over unit GROUND area (remark: typical example is snow load, but it is an acceptable* approximation *for the dead load of the shell, if it is shallow (f<<a, b)*
- *the ground area is rectangular*
- *no other load is posed on the structure*

see Lecture notes for future reference.

According to the membrane theory of shells, loads are equilibrated by forces (normal and shear) acting in the tangent plane of the surface. This special type of loading (p_z, see above) in quadrilateral shells can be equilibrated by shear forces alone. This leads to a very important architectural insight: the shell can be supported by semi-rigid edges (taking only shear forces), ie. edge-beams along its perimeter*
**We refer to the Lecture notes here again, for further study: actually, in real-life problems, for loads are never 'ideal', two edges should provide full support (taking shear and normal forces aswell). See Figures how this is implemented architecturally (combination of quadrilateral shells into a bigger roof structure, where the neighbouring shells provide the necessary lateral support)*







It is customary in shell theory, to consider the horizontal projection of forces aswell. Hence the following notation is used below:
N_{xy} for shear force
$\overline{N_{xy}}$ for its horizontal projection
N_{xy}^V for its vertical component (if applicable)
Note, that by *force* resultant stress is meant, summed over the thickness of the shell [kN/m]. Also note, that for the load is considered to be distributed over unit ground area, the unit length is also measured on the horizontal plane (even if the edge is inclined).

Internal forces are determined based on equilibrium equations:
Vertical equilibrium: The loads of the shell are equilibrated by the edge beams (*shear force of the shell causes normal force in the edge beam*), which transfer those to the supports. In order for the structure to be in equilibrium, the load must be equilibrated by the resultant of the vertical components of the shear forces in the shell (*ie, normal force in edge beam*) and by extension, the vertical support reactions.

Reciprocity of shear forces/ see also Complementary shear stress: Another key concept in the derivation of the results below is the reciprocity of shear forces (*Reference: Strength1*) which only holds in the horizontal plane, for forces over unit length. This results the second guiding equation of equilibrium:

$$\overline{N_{xy}}_a = \overline{N_{xy}}_b$$

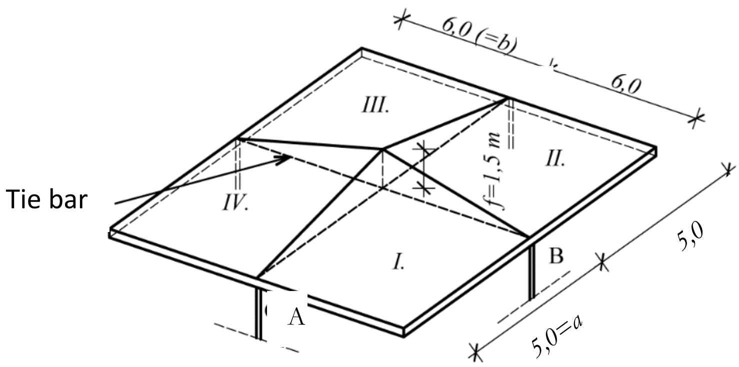
T3/11 INTERNAL FORCE DISTRIBUTION OF AN (INVERTED) UMBRELLA ROOF

Data

self-weight: g_k=1,5 kN/m^2

snow: s_k=1,5 kN/m^2

Both over unit ground area



Load analysis

The safety factors are

$\gamma_q = 1{,}5$ (live load) and $\gamma_g = 1{,}35$ (self-load).

Hence, the design values of the loads:

$g_d = g_k\gamma_g$=1,35·1,5=2,025 kN/m^2

$s_d = s_k\gamma_q$=1,5·1,5=2,25 kN/m^2

The total load of the structure is:

$$P_{z\_Ed} = g_d + s_d = \mathbf{4{,}28\ kN/m^2}$$

(a) Determine the shear force acting in the shell!

As seen in the theory part, we have to equations of equilibrium

Reciprocity of shear forces:

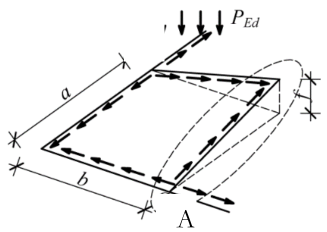
$$\overline{N_{xy}}_a = \overline{N_{xy}}_b = \overline{N_{xy}}$$

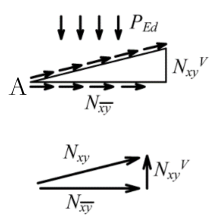
and vertical equilibrium:

$$(N_{xya}{}^V * a + N_{xyb}{}^V * b) = (N_{Eda}^{column}/2 + N_{Edb}^{column}/2) = p_{Ed}ab$$

From geometry:

$$\overline{N_{xy}}_a * \frac{f}{a} = N_{xya}^V \text{ and } \overline{N_{xy}}_b * \frac{f}{b} = N_{xyb}^V$$





hence

$$\overline{N_{xy}} * f * 2 = p_{Ed}ab = 4{,}28 * 5 * 6 = \mathbf{128{,}4\ kN}$$

$$\overline{N_{xy}} = \frac{p_{Ed}ab}{2f} = \frac{128{,}4}{2 * 1{,}5} = \mathbf{42{,}8\ kN/m}$$

$$N_{xya}^V = \frac{p_{Ed}ab}{2f} * \frac{f}{a} = \frac{p_{Ed}b}{2} = \frac{4{,}28 * 6}{2} = \mathbf{12{,}84\frac{kN}{m}};\ N_{xyb}^V = \frac{p_{Ed}ab}{2f} * \frac{f}{b} = \frac{p_{Ed}a}{2} = \frac{4{,}28 * 5}{2} = \mathbf{10{,}7\frac{kN}{m}}$$

$$N_{xya} = \sqrt{N_{xya}^V{}^2 + \overline{N_{xy}}^2} = \mathbf{44{,}7\frac{kN}{m}};\ N_{xyb} = \sqrt{N_{xyb}^V{}^2 + \overline{N_{xy}}^2} = \mathbf{44{,}1\frac{kN}{m}}$$

(b) Determine the vertical support reaction forces!

At support A, the vertical reaction force is (*there are two parallel edges!*)

$$A_V = 2N_{xya}^V a = 2 * 12{,}84 * 5 = 128{,}4\ kN$$

At support B:

$$B_V = 2N_{xyb}^V b = 2 * 10{,}7 * 6 = 128{,}4\ kN*$$

**note, that this could have also been derived directly, using the concept of loading area: each shall support ¼ of the shell.*

(c) Determine the maximal normal force acting in the edge beams!

Inclined edge beam parallel to side *a*:

$$N_{edgeai} = N_{xya} * a = 44{,}7 * 5 = \mathbf{223{,}5\ kN}$$

Straight edge beam side a:

$$N_{edgea} = \overline{N_{xy}} * a = 42{,}8 * 5 = \mathbf{214\ kN}$$

Inclined edge beam parallel to side *b*:

$$N_{edgebi} = N_{xyb} * b = 44{,}1 * 6 = \mathbf{264{,}6\ kN}$$

Straight edge beam side a:

$$N_{edgeb} = \overline{N_{xy}} * b = 42{,}8 * 6 = \mathbf{256{,}8\ kN}$$

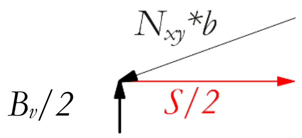
(d) Determine the forces acting in the tie bars:

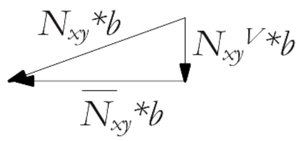
From the equilibrium equations at support A:

$S_a = 2 \cdot \overline{N_{xy}} \cdot a = 2 \cdot 42{,}8 \cdot 5{,}0 = \mathbf{428\ kN}$

From the equilibrium equations at support B:

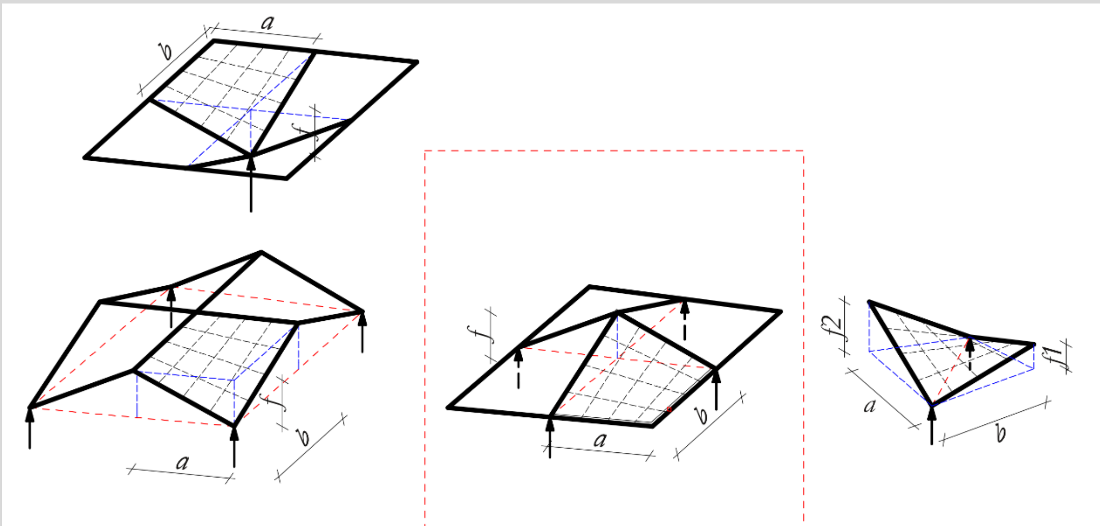
$S_b = 2 \cdot \overline{N_{xy}} \cdot b = 2 \cdot 42{,}8 \cdot 6{,}0 = \mathbf{513{,}6\ kN}$





THEORY

Typical arrangements of quadrilateral shells



2 inclined edges, 1 vertical support

2 inclined edges, 2 vertical supports

4 inclined edges, 2 vertical supports

Both internal forces and support reaction forces can be determined based on the method shown above, even, if *a* differs from *b*. Hence each can be expected at TEST 2.

See study aid for further reference, including specific formula and extra exercise T31.