Lecture 2a Systems of equations,

How to solve a system of 2 linear equations?

a,b,c,d,e,f are known, find x,y!

(1) $ax+by=c$
(2) $dx+ey=f$

Express one of the variables from one equation, e.g. x from (1):

(3) $x=(c-by)/a$

Substitute (3) into the remaining equation (2) and express the unknown:

(4) $d/a*(c-by)+ey=f$
(5) $y=(f-cd/a)/(e-bd/a)=(fa-cd)/(ea-bd)$

Now you can use (3) to find the other unkown

3 or more equations:

same procedure: express variables 1-by-1 until you end up having one variable in one equation

Excercise 1: solve yourself

$x+3y=4$

$-2x+y=0$

do not use the formulas above,do the calculations yourself!

Try expressing x first, and expressing y first! You will get the same result

(Result: $x=4/7$; $y=8/7$)

Excercise 2: solve yourself

$x+y=2$

$x+z=4$

$y+z=3$

(Result: $x=3/2$; $y=1/2$;$z=5/2$)

Excercise 3: solve

(1) $x+2y=2$
(2) $2x+4y=1$

Solution: from (1) we have

(3) $x=2-2y$

From (2) and (3) we have

4-4y+4y=1

which is contradiction: this system of equations has no solution!

How can you make sure that a system of linear equations has exactly one solution?

Condition 1: number of equations must be = number of unkonwns
Condition 2: if you write the system of equations in matrix form then the determinant of the matrix must be non-zero

Excercise 3 in matrix format:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

The determinant of the matrix is 1*4-2*2=0

Lecture 2b: equilibrium of point mass

Newton's laws:

1) An object remains at rest or moves at constant velocity unless it is acted upon by an external force
2) if it is acted upon by a force then F=ma
3) When a body exerts force to a second body, then the second one also exerts a force of opposite direction and equal magnitude to the first one

We also use another law:

4) The combined effect of 2 or more simultaneously acting forces is the same as the effect of their vector sum.

Law 1) implies equilibrium if no force – this is rare

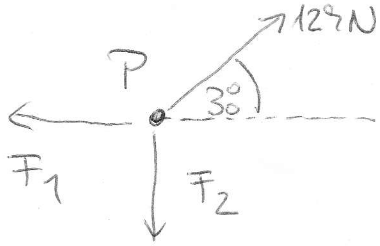
Law 2) implies no equilibrium of there is one force not e~~w~~qual to zero. But what happens if there are more than 1?

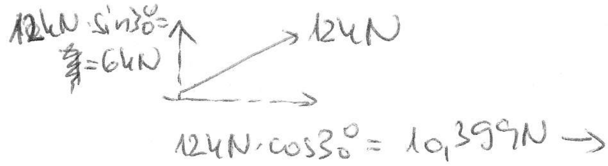
Law 4): more than 1 forces can be replaced by 1 force

Law 3): forces inside a structure come in pairs which will be a great help for us.

In the~~o~~ir original form, Newton's laws apply only to point-like bodies, or to bodies where the line of action of every force goes through the same point.

Excercise 1: find the magnitude of F_1 and F_2 if the point mass is in equilibrium

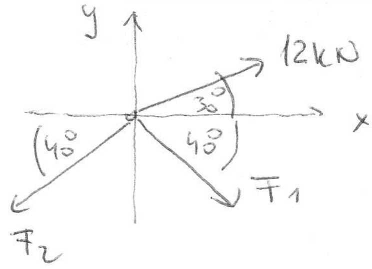


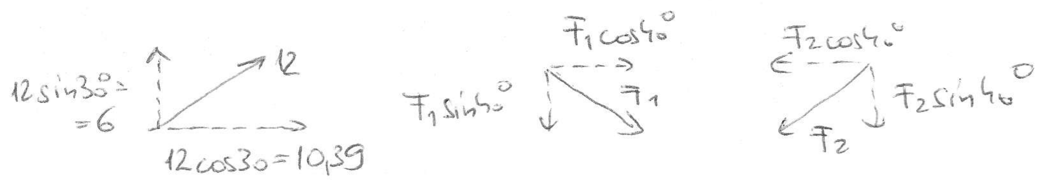


$\Sigma F_x = 10{,}39kN - F_1 = \emptyset \rightarrow \boxed{F_1 = 10{,}39kN \leftarrow}$

$\Sigma F_y = 6kN - F_2 = \emptyset \rightarrow \boxed{F_2 = 6kN \downarrow}$

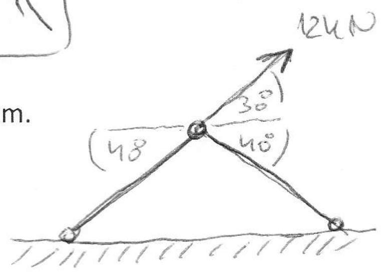
Excercise 2: find F_1 and F_2 if the point mass is in equilibrium





$$\left.\begin{aligned} \Sigma F_x &= 10{,}39 + 0{,}766 F_1 - 0{,}766 F_2 = \emptyset \\ \Sigma F_y &= 6 - 0{,}6428 F_1 - 0{,}6428 F_2 = \emptyset \end{aligned}\right\} \text{system of 2 equations}$$

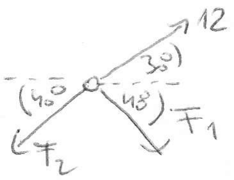
solution: $\boxed{F_2 = 11{,}45 \swarrow}$ $\boxed{F_1 = -2{,}11 \nwarrow}$



Excercise 3: find the forces acting in the cables if the system is in equilibrium.

In this excercise you have to examine the equilibrium of point P.

First make a „free-body diagram": a drawing of P with all forces



Now this excercise is the same as excercise 2 so we do not solve it again.

(note however that $F_1 = -2{,}11$ is impossible: a cable cannot be compressed)

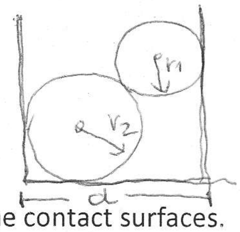
Excercise 4: Find F and α if the point mass is in equilibrium

solve yourself!

answer: $F = 21{,}79$

$\alpha = 216{,}6°$

Excercise 5: the surfaces of the container and the cylinders are frictionless. Find the forces acting between the two cylinders and the forces acting between the cylinders and the walls of the cuboid-shaped container. r_1=10 cm; r_2 = 20 cm; d=50 cm; masses of cylinders m_1=1 kg; m_2=8 kg.



Weight of cylinders: $G_1=m_1g=10N$ $G_2=m_2g=80N$

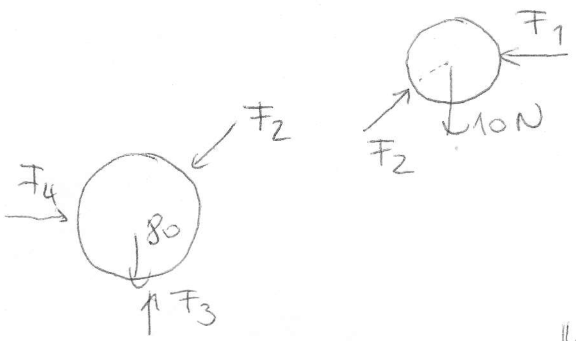
We will use the fact that **both** cylinders are in equilibrium.

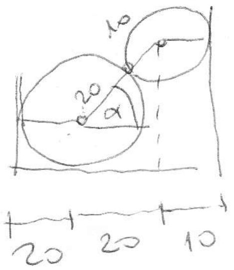
Frictionless surface means that contact forces are perpendicular to the contact surfaces.

Every force acting at a cylinder goes through the middle of the cyclinder so we can use Newton's laws.

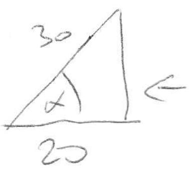
We use law 3) for the force acting between the two cylinders

As always, first we draw free body diagrams for both cylinders:





Then we write equations of equilibrium:



$\leftarrow \alpha = 48{,}1°$

$\Sigma F_x small = F_1 - F_2\cos\alpha = \phi \longrightarrow \boxed{F_1 = F_2\cos\alpha = 8{,}88\ kN}$

$\Sigma F_y small =$ ~~[illegible]~~

$10 - F_2 \sin\alpha = \phi \rightarrow F_2 = 13{,}37\ kN$

$\Sigma F_x big = F_4 - F_2\cos\alpha = \phi \rightarrow \boxed{F_4 = 8{,}88\ kN}$

$\Sigma F_y big = 80 + F_2 \sin\alpha - F_3 = \phi \rightarrow \boxed{F_3 = 90\ kN \uparrow}$