

Mathematical basics: vectors, equation systems

1. Given vectors $\mathbf{a}=[1;1]$, $\mathbf{b}=[0;-5]$ and $\mathbf{c}=[-3;7]$. Determine the vector $\mathbf{d}=4\mathbf{a}+3\mathbf{b}-\mathbf{c}$ and its length!
2. Given a parallelogram ABCD: A(1;3), B(4;7), C(2;8), D(-1;4). What are the distances between the opposite vertices?
3. Given a line segment AB: A(1;1), B(7;5), which is divided to 3 equal part by points C and D. Determine the coordinates of points C and D!
4. Break up vector $\mathbf{a}=[-2;6]$ into components, which are parallel to the coordinate axes!
5. Determine the scalar product of vectors $\mathbf{a}=[-2;-5]$ and $\mathbf{b}=[3;3]$! Also determine the angle between the two vectors! Determine the magnitude of the vector product (cross product) assigned to the vectors \mathbf{a} and \mathbf{b} if \mathbf{a} is added as 3rd coordinate! Using this latter, calculate the angle between vectors \mathbf{a} and \mathbf{b} !
6. Given a triangle ABC: A(2,5), B(10,3), C(5,8). Determine the angle at vertex B (solve the problem by using the scalar product)! Determine the area of the triangle!
7. How much is α , if vectors $\mathbf{a}=[2;6]$ and $\mathbf{b}=[3;\alpha]$ are perpendicular to each other (use the scalar product)?
8. Given three vertices of the parallelogram ABCD: A(0;2), B(4;-3), C(2;-7). Determine the area of the parallelogram (use the vector product for the solution)! Determine the coordinates of the fourth vertex D!
9. Determine the orthogonal projection of vector $\mathbf{a}=[-2;-5]$ to a line which closes 45° angle with axis x !
10. Solve the following system of equations!
$$-10x + 3y = 25$$
$$5x - 4y = -2$$
11. Break up vector $\mathbf{a}=[5;4]$ into the sum of 2 vectors, one of which is parallel to $\mathbf{v}_1=[1;3]$ and the other one is parallel to $\mathbf{v}_2=[3;-1]$! (You have to solve a system of equations.)