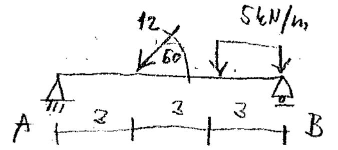
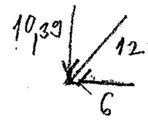
Belső erők

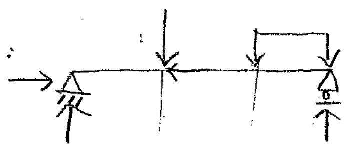
4-1.)

a.)





reakciók:



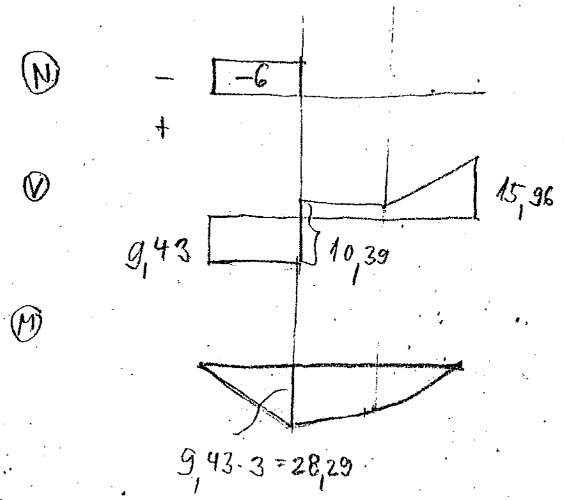
$\Sigma M_A = 0$

$3 \cdot 12 \cdot \sin 60° + 5 \cdot 3 \cdot 4{,}5 - B_y \cdot 9 = 0$

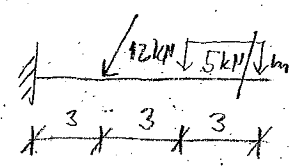
$B_y = 15{,}96 \text{ kN}$

$A_y = 12 \cdot \sin 60° + 5 \cdot 3 - 15{,}96 = 9{,}432 \text{ kN}$

$A_x = 6 \text{ kN}$



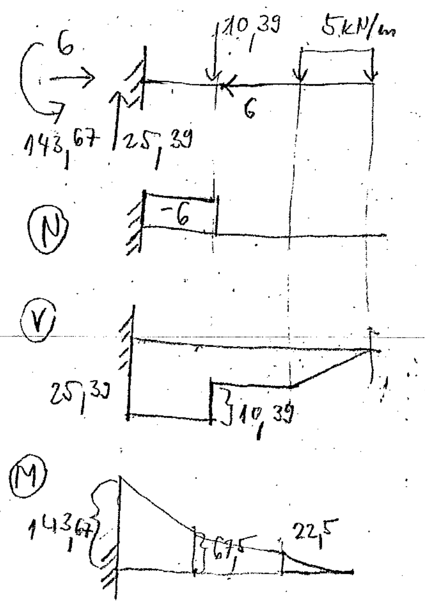
b.)



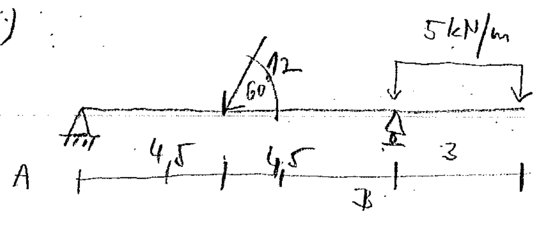
$A_y = 12 \cdot \sin 60° + 5 \cdot 3 = 25{,}39 \text{ kN}$

$A_x = 12 \cdot \cos 60° = 6 \text{ kN}$

$M_A = 12 \cdot \sin 60° \cdot 3 + 5 \cdot 3 \cdot 7{,}5 = 143{,}67 \text{ kNm}$



c.)

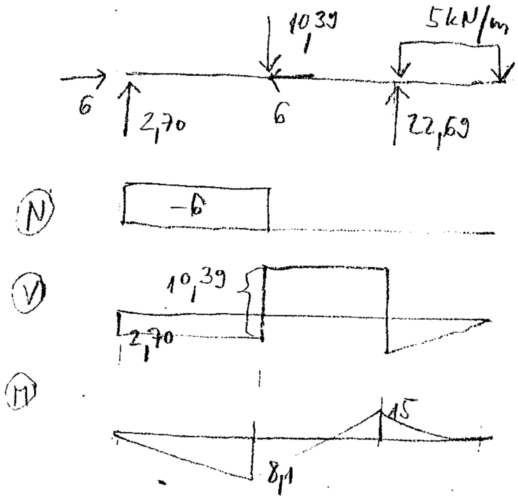


$\Sigma M_A = 0 \qquad 12 \cdot \sin 60° \cdot 4{,}5 + 5 \cdot 3 \cdot 10{,}5 - B_y \cdot 9 = 0$

$B_y = 22{,}69$

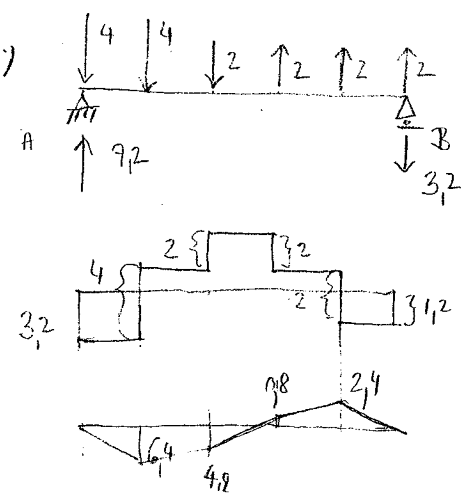
$A_y = 12 \cdot \sin 60 + 5 \cdot 3 - 22{,}69 = 2{,}70$

$A_x = 6$



4-2)

a)



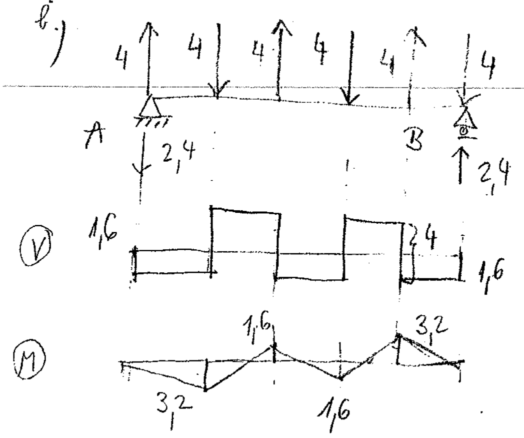
$\Sigma M_A = 0$

$4 \cdot 2 + 2 \cdot 4 - 2 \cdot (6 + 8 + 10) + B_y \cdot 10 = 0$

$B_y = 3{,}2$

$A_y = 7{,}2$

b)

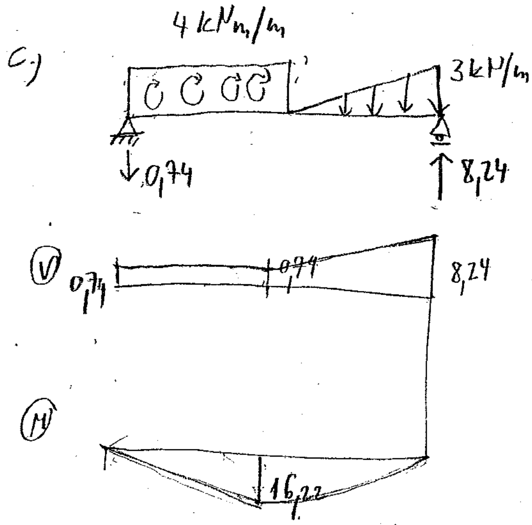


$\Sigma M_A = 0 \quad 4 \cdot (2 + 6 + 10) - 4 \cdot (4 + 8) - B_y \cdot 10 = 0$

$B_y = 2{,}4$

$A_y = 2{,}4$

c.)

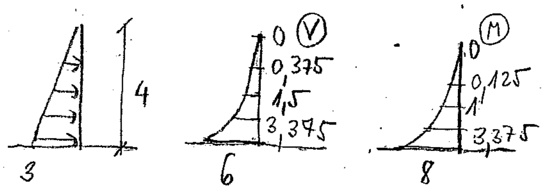


$\Sigma M_A = 0 \quad 4\cdot 5 + \frac{3\cdot 5}{2}\cdot 8{,}33 - B_y \cdot 10 = 0$

$B_y = 8{,}24$

$A_y = \frac{3\cdot 5}{2} - 8{,}24 = -0{,}74$

4-4.) a.)

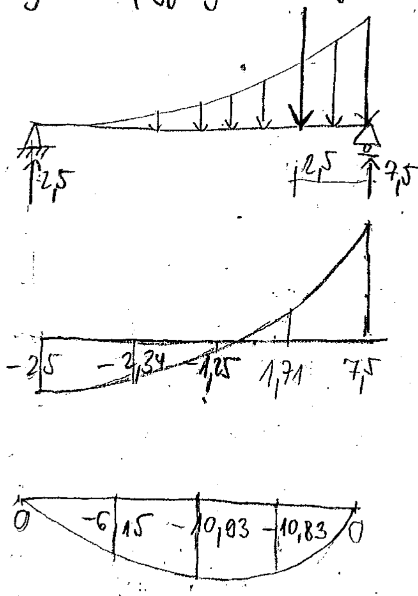


$\int \frac{3}{4}\cdot x\,dx = \frac{3}{4}\cdot\frac{x^2}{2} = \frac{3}{8}\cdot x^2 \to V$

$\int \frac{3}{8}\cdot x^2 dx = \frac{3}{8}\cdot\frac{x^3}{3} = \frac{x^3}{8} \to M$

A nyomatékfüggvény deriváltja a nyíróerő, a nyíróerő deriváltja a teher.

b.)



$p = \frac{3}{100}\cdot x^2$

eredő: $R = 10$ (első feladat alapján)

$\to A_y = 2{,}5$

$B_y = 7{,}5$

$V = \int \frac{3x^2}{100}dx = \frac{3}{100}\cdot\frac{x^3}{3} + C_1 = \frac{x^3}{100} + C_1$

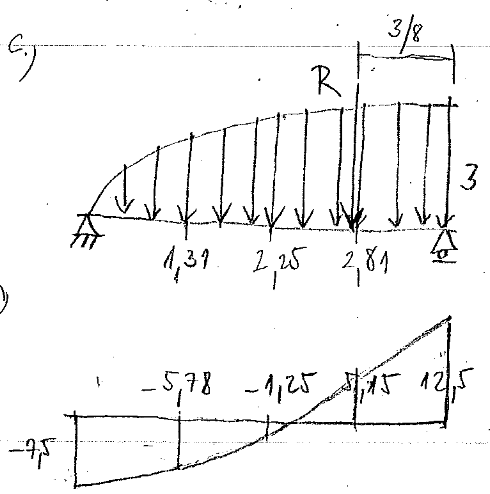
$A = 2{,}5 \to C_1 = -2{,}5$

$\int \frac{x^3}{100} + 2{,}5\,dx = \frac{x^4}{400} - 2{,}5x + C_2$

B pontban 0 kell legyen:

$\frac{10^4}{400} - 2{,}5\cdot 10 + C_2 = 0 \to C_2 = 0$

c.)



$p = 3 - \frac{3\cdot(x-10)^2}{100}$

$R = 20 \to A_y = 7{,}5$

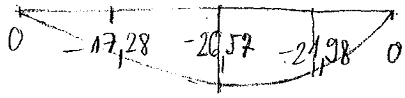
$B_y = 12{,}5$

$\int 3 - \frac{3\cdot(x-10)^2}{100}dx = \int\left(\cancel{3} - \frac{3x^2}{100} + \frac{6x}{10} - \cancel{3}\right)dx =$

$= -\frac{\cancel{3}}{100}\cdot\frac{x^3}{\cancel{3}} + \frac{6}{10}\cdot\frac{x^2}{2} + C_1 = -\frac{x^3}{100} + \frac{3}{10}\cdot x^2 + C_1$

$A_y = 7{,}5 \to C_1 = -7{,}5$

(M)



$$\int\left(-\frac{x^3}{100}+\frac{3}{10}\cdot x^2-7{,}5\right)dx$$

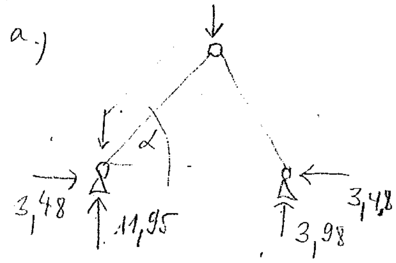
$$=-\frac{1}{100}\cdot\frac{x^4}{4}+\frac{3}{10}\cdot\frac{x^3}{3}-7{,}5\,x+C_2=-\frac{x^4}{400}+\frac{x^3}{10}-7{,}5x$$

B pontban: $-\frac{x^4}{400}+\frac{x^3}{10}-7{,}5x+C_2=0$

$C_2=0$

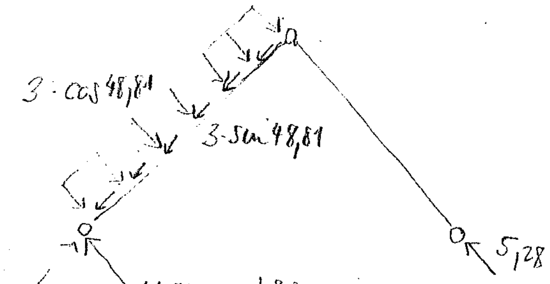
4-6.) A reakcióerőket már az első részben kiszámoltuk.

a.)



Az erőket felbontjuk a rúd tengelyével párhuzamos és merőleges komponensekre!

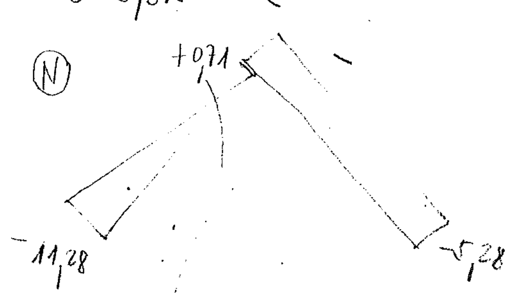
$\alpha = 48{,}81°$ $\quad l = 5{,}315$



$11{,}95\cdot\cos 48{,}81 - 3{,}48\cdot\sin 48{,}81 = 5{,}25$

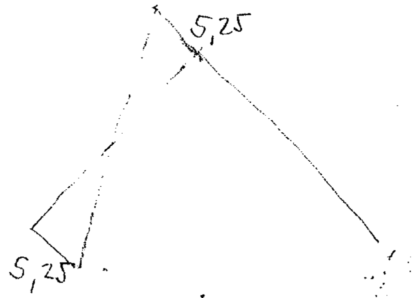
$11{,}95\cdot\sin 48{,}81 + 3{,}48\cdot\cos 48{,}81 = 11{,}28$

(N)

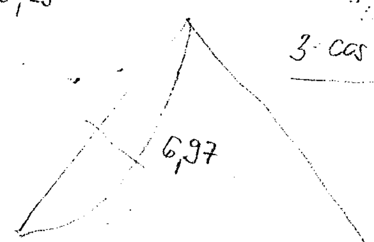


$-11{,}28 + 5{,}315\cdot 3\cdot\sin 48{,}81$

(V)

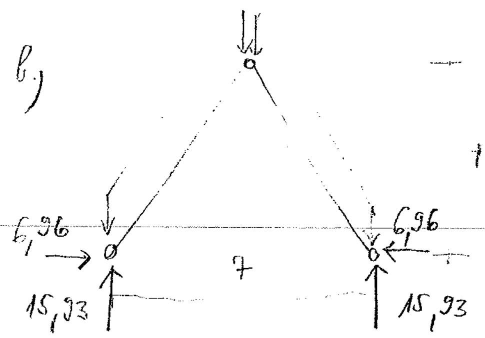


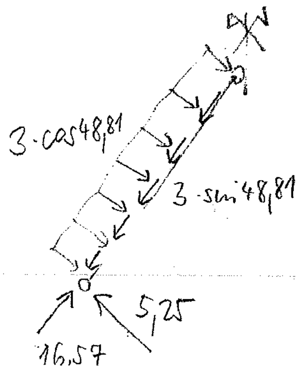
(M)



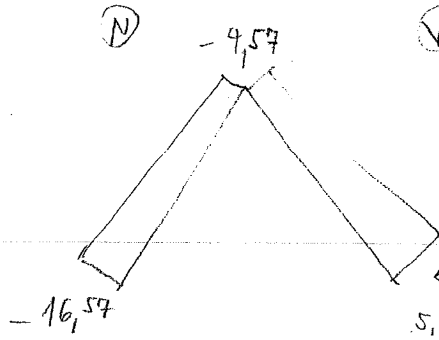
$$\frac{3\cdot\cos 48{,}81\cdot 5{,}315^2}{8}=$$
$$=6{,}97$$

b.)

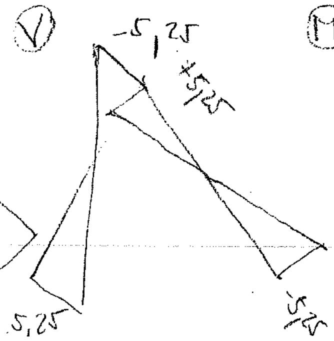




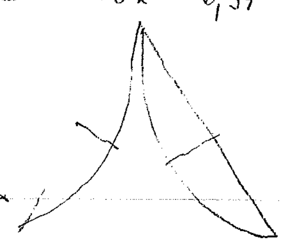
(N)



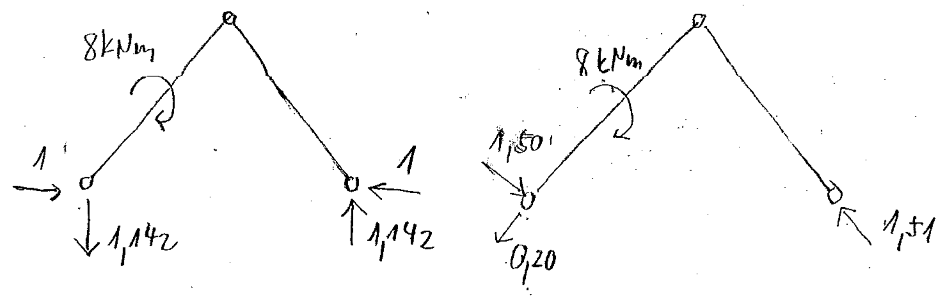
(V)

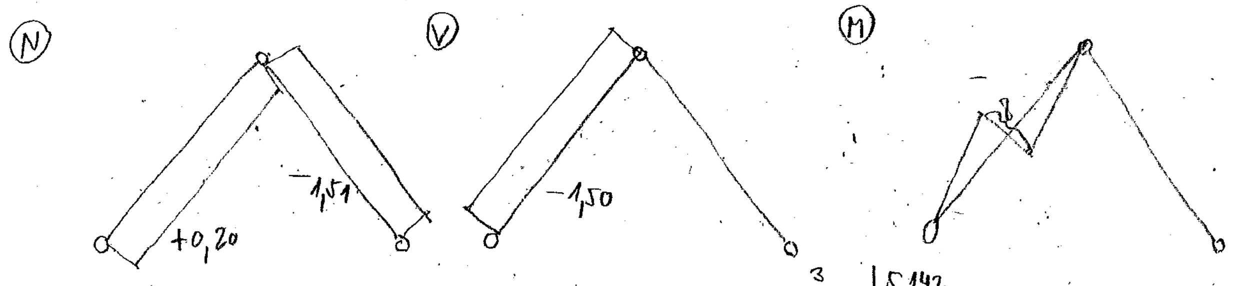


(M) $M_{max} = 6{,}97$

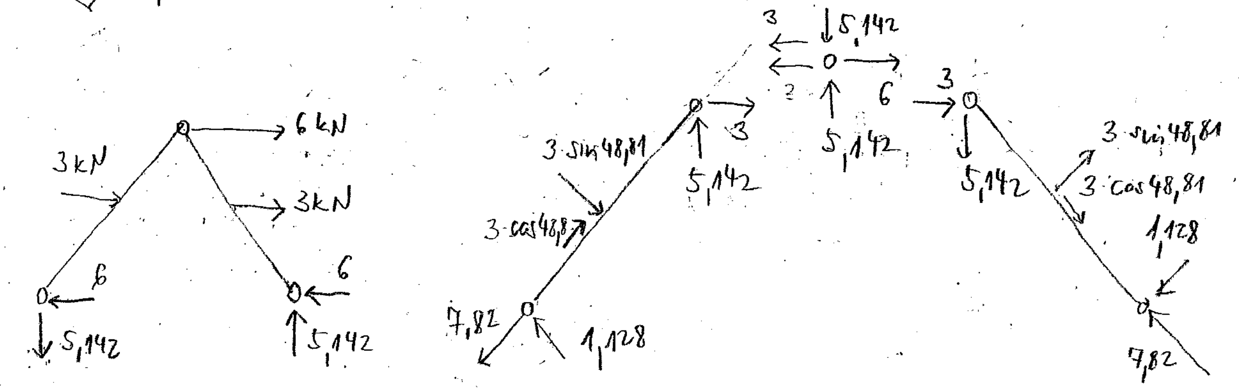


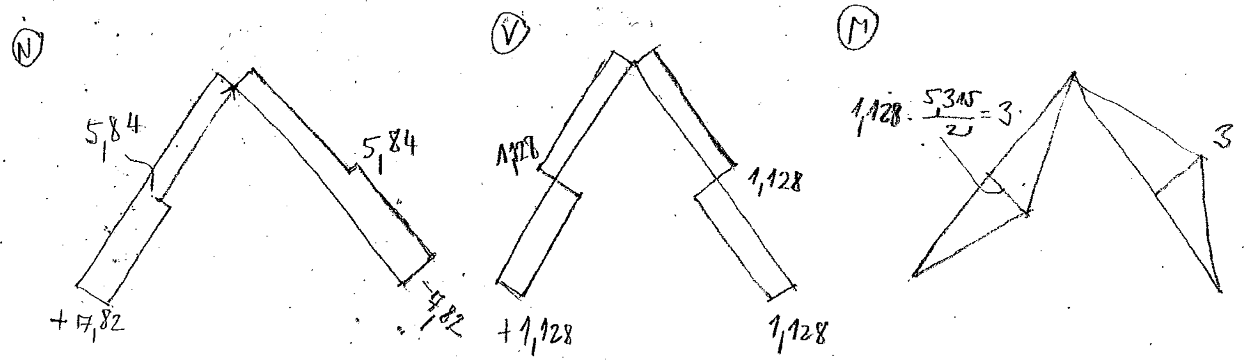
c)


8kNm
1
1
1,142
1,142
8kNm
1,50
0,20
1,51

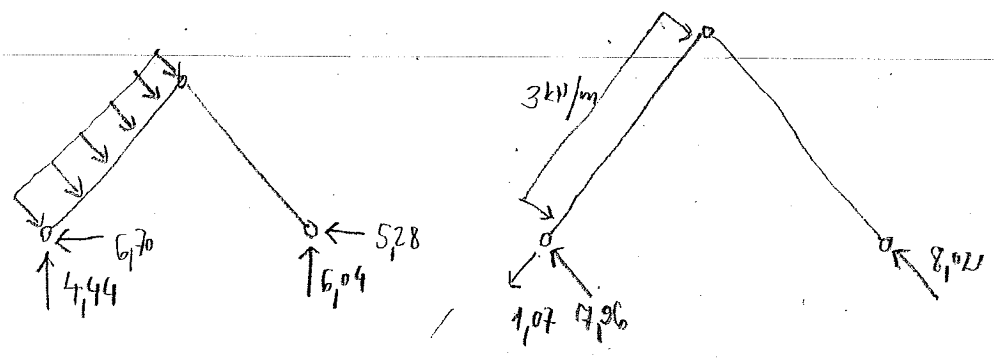

N
+0,20
-1,51
V
-1,50
M
8

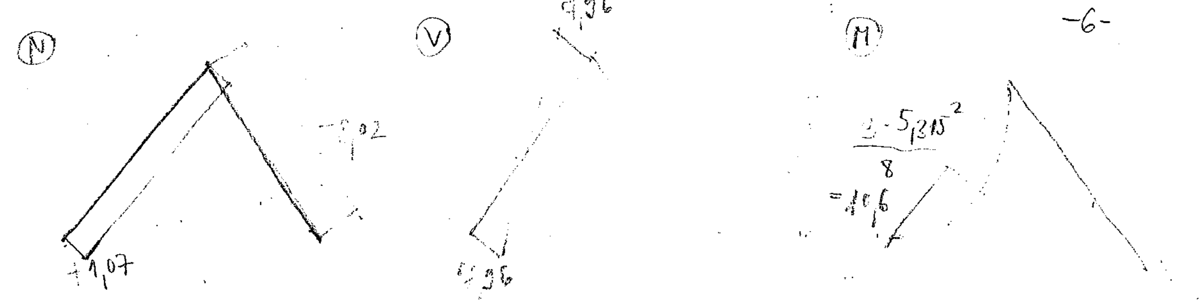
d)


6kN
3kN
3kN
6
6
5,142
5,142
3 sin48,81
3 cos48,81
3
5,142
7,82
1,128
5,142
3
3
6
5,142
3
5,142
3 sin48,81
3 cos48,81
1,128
7,82

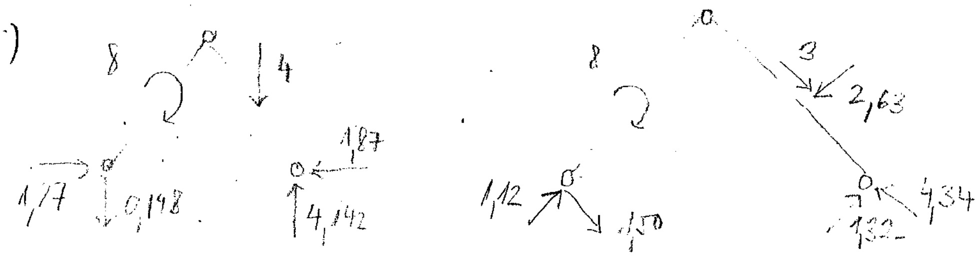

N
5,84
5,84
+7,82
-7,82
V
1,128
1,128
+1,128
1,128
M
1,128 · 5,315/2 = 3
3

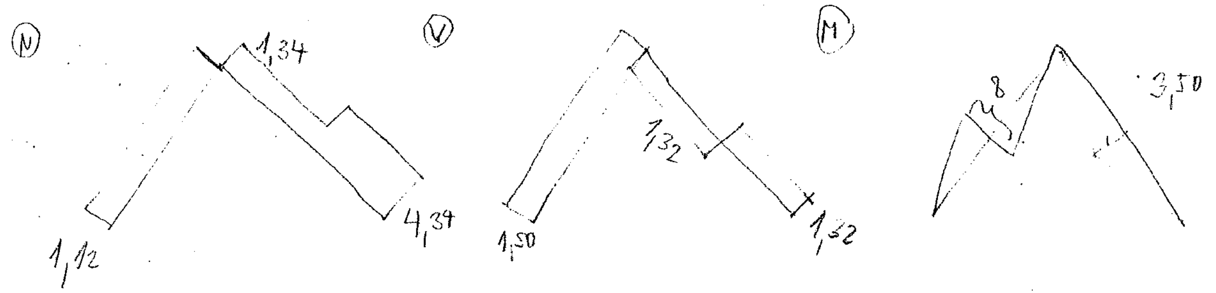
e)


6,70
4,44
5,28
6,04
3kN/m
1,07
7,96
8,02



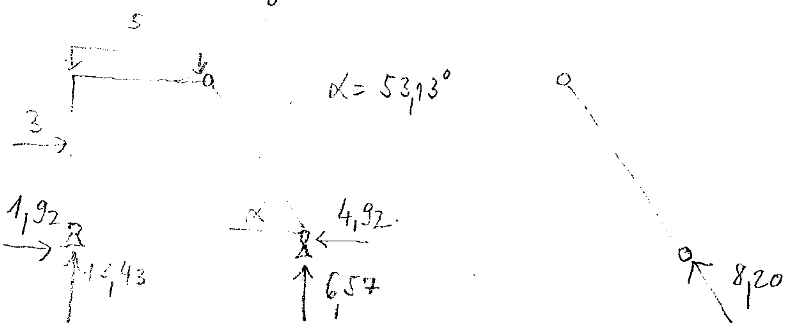
f.)

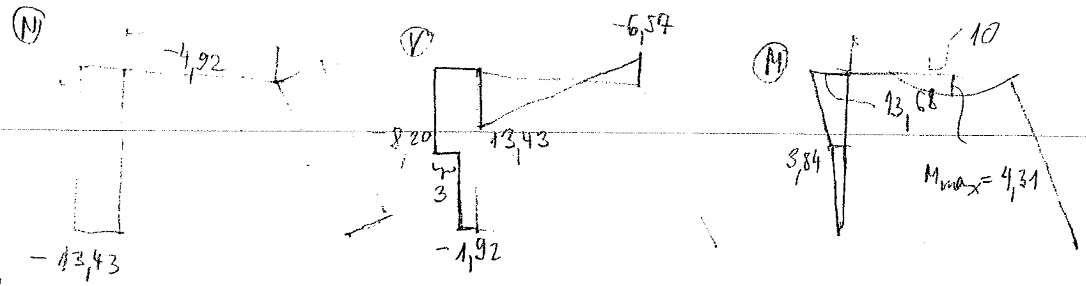




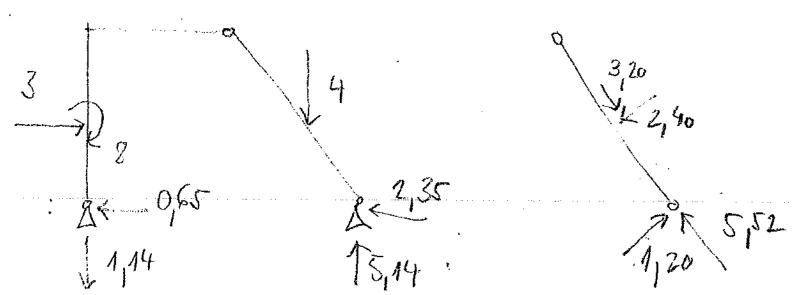
4.7.) A reakciókat már meghatároztuk az előző részben!

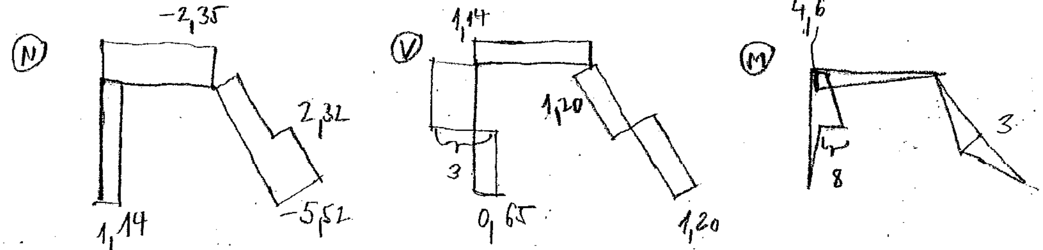
a.)





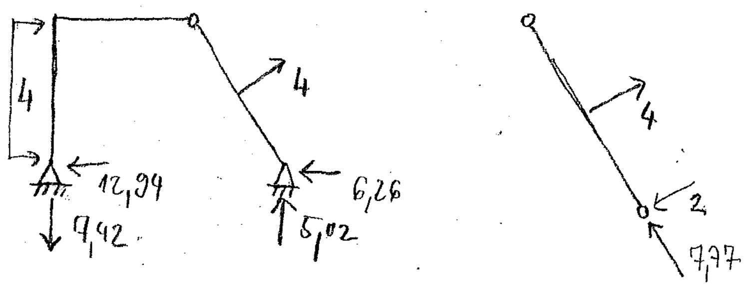
b.)

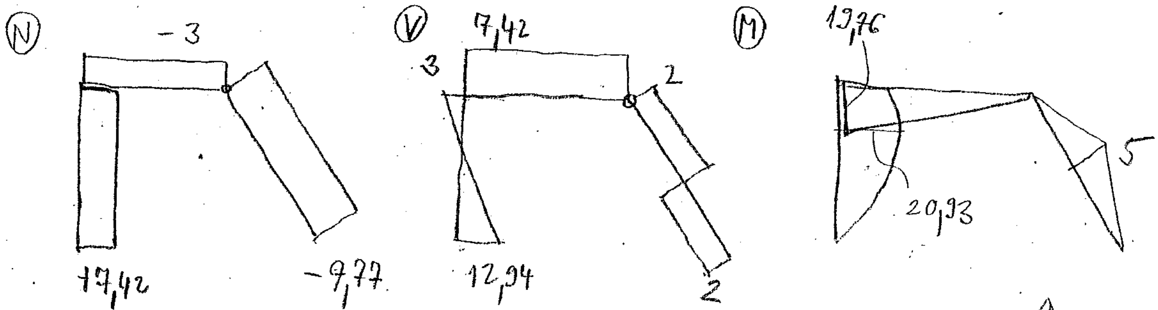




$0{,}65 \cdot 4 + 8 - 3 \cdot 2 = 4{,}6$

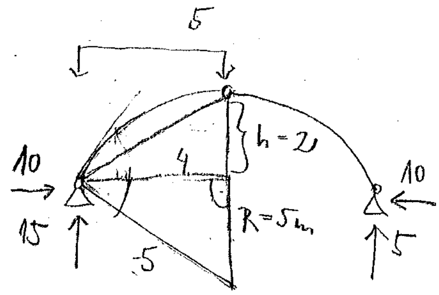
a)





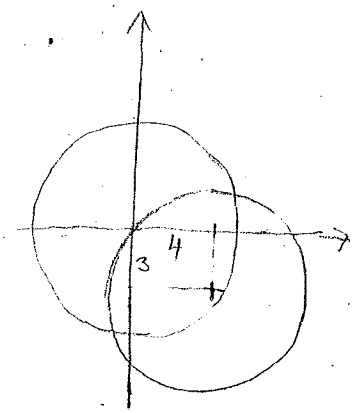
4-9)

a)



$(x-4)^2 + (y+3)^2 = r^2$

$y = \sqrt{r^2 - (x-4)^2} - 3 \rightarrow h$

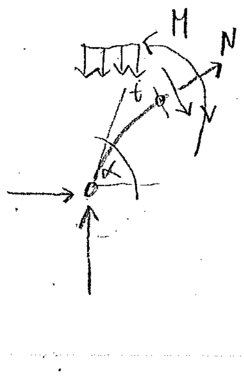


$\Sigma M_A = 0 \qquad 5 \cdot 4 \cdot 2 - B_y \cdot 8 = 0$

$B_y = 5 \quad A_y = 15$

$\Sigma M_{C\,bal} = 0 \qquad 15 \cdot 4 - 5 \cdot 4 \cdot 2 - A_x \cdot 2 = 0$

$A_x = 10 \quad B_x = 10$



$N + A_y \cdot \sin\alpha + A_x \cdot \cos\alpha - p \cdot x \cdot \sin\alpha = 0$

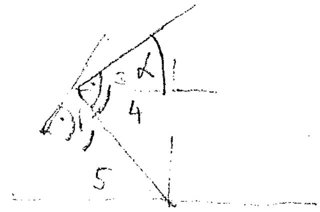
$N = 5 \cdot x \cdot \sin\alpha - 15 \cdot \sin\alpha - 10 \cdot \cos\alpha$

$V - A_y \cdot \cos\alpha + A_x \cdot \sin\alpha + p \cdot x \cdot \cos\alpha = 0$

$V = 15 \cdot \cos\alpha - 10 \cdot \sin\alpha - 5 \cdot x \cdot \cos\alpha$

$M + A_x \cdot h - A_y \cdot x + p \cdot x \cdot \frac{x}{2} = 0$

$M = 15 \cdot x - 10 \cdot h - 5 \cdot x^2$



$\alpha_1 = 53{,}14°$ $h_1 = 0$

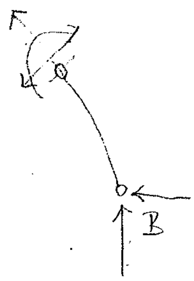
$\alpha_2 = 36{,}87°$ $h_2 = 1$

$\alpha_3 = 23{,}58°$ $h_3 = 1{,}58$

$\alpha_4 = 11{,}54°$ $h_4 = 1{,}89$

$\alpha_5 = 0°$ $h_5 = 2$

x	0	1	2	3	4
h	0	1	1,58	1,89	2
α	53,14°	36,87°	23,58°	11,54°	0°
N	−18	−14	−11,16	−9,79	−10
V	+1	+2	+0,582	−2	−5
M	0	+2,5	+4,2	+3,6	0

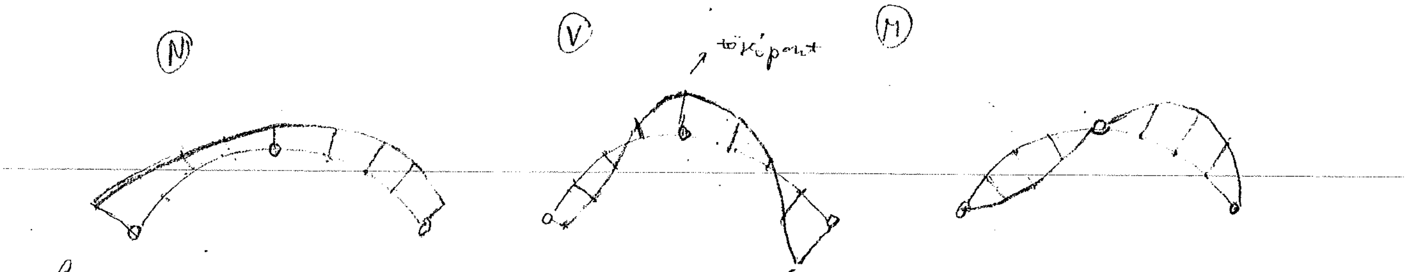


$N = -5 \cdot \sin\alpha - 10 \cdot \cos\alpha$

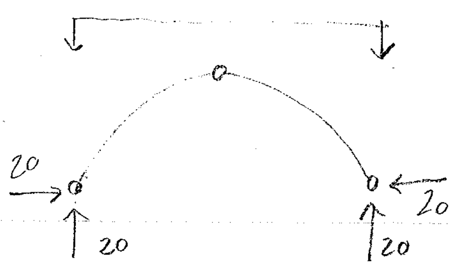
$V = 5 \cdot \cos\alpha - 10 \cdot \sin\alpha$

$M = 5 \cdot x - 10 \cdot h$

N	−10	−11	−11,16	−10,79	−10
V	−5	−2	+0,58	+2,89	+5
M	0	−5	−5,8	−3,9	0



6,



$\Sigma M_{c\,bal} = 0 \quad 20 \cdot 4 - 5 \cdot 4 \cdot 2 - A_x \cdot 2 = 0$

$A_x = 20 \quad B_x = 20$

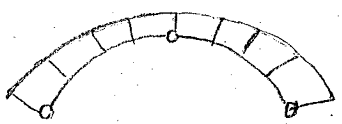
$$N = 5 \cdot x \cdot \sin\alpha - 20 \cdot \sin\alpha - 20 \cdot \cos\alpha$$

$$V = 20 \cdot \cos\alpha - 20 \cdot \sin\alpha - 5 \cdot x \cdot \cos\alpha$$

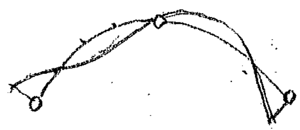
$$M = 20 \cdot x - 20 \cdot h - \frac{5}{2} \cdot x^2$$

N	−28	−25	−22,33	−20,6	−20
V	−4	0	+1,16	+0,89	0
M	0	−2,5	−1,6	−0,3	0

N

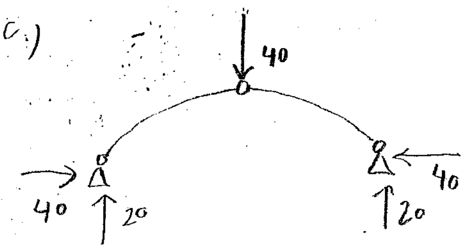


V



M

c)



$\Sigma M_{c\,bal} = 0 \qquad 20 \cdot 4 - A_x \cdot 2 = 0$

$A_x = B_x = 40$

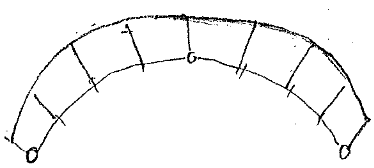
$$N = -20 \cdot \sin\alpha - 40 \cdot \cos\alpha$$

$$V = 20 \cdot \cos\alpha - 40 \cdot \sin\alpha$$

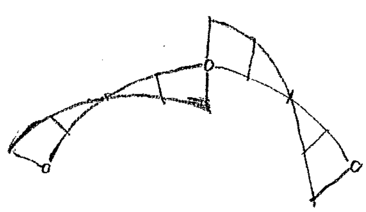
$$M = 20 \cdot x - 40 \cdot h$$

N	−40	−44	−44,66	−43,19	−40
V	−20	−8	+2,32	+11,6	+20
M	0	−20	−23,2	−15,6	0

N



V



M