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NEW ELASTIC PHENOMENOLOGICAL MATERIAL LAW FOR TEXTILE COMPOSITE

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Summary. A new phenomenological material law for textile composites is described in the paper. The new material law can follow the nonlinear behavior of the material and the contraction of the orthogonal yarns.

1 INTRODUCTION

The most recent methods to handle the nonlinear behavior are the micro models[2,3,4,8]. There are methods to describe the nonlinear behavior by nonlinear functions, for example the dense net method[1,6], which can not handle the connection between the two orthogonal yarn direction. The spline methods[5,7] can describe the surface of a stress as a function of the two orthogonal elongation, but the spline functions can work only for interpolation.

The new model described here is a phenomenological model. Exponential functions are used to fit the experimental stress-strain data for bi-directional stress state.

There are two basic problems: i., describe real stress-elongation curve with functions, ii., describe the bi-directional behavior satisfying the law of energy conservation.

To solve the i. problem practical to use a higher order polynomial functions. But they are valid only in the interval of the measurements. Out of this interval the polynomial can lead to numerical divergence in the highly nonlinear analysis of a membrane structure. It is better to choose asymptotic functions, and they are enough to describe the stress-strain diagram well in the service load level.

To solve the ii. problem a special function is needed with the two orthogonal elongations as variable. This functions will be described in the 3. section.

2 ONE DIRECTIONAL LOADING

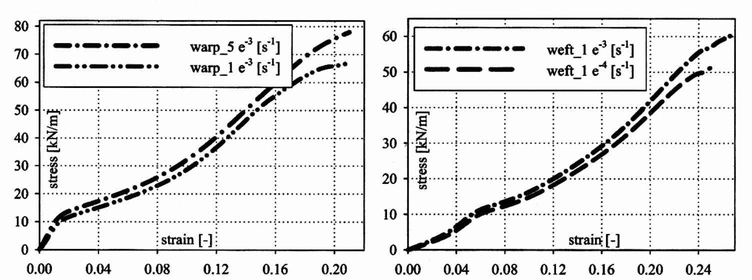

Figure 1: Typical stress-strain diagrams[1]

Two typical one directional loading test curve is shown in Figure 1 in the warp and in the fill direction. The typical curve in the service load level can be divided into two part with different slope. With a combination of two asymptotic exponential functions the one directional behavior of the textiles can be described very well:

$$\sigma_w = a_1 \varepsilon_w \left(1 - e^{-\varepsilon_w^2}\right) + a_2 e^{-\varepsilon_w^2},\ if\ \sigma_f = 0 \tag{1}$$

where σ_w is the stress in the warp direction, ε_w is the elongation in the warp direction and a_1, a_2 are parameters, and σ_f is the stress in the fill direction. This function fit well to a real stress-strain diagram, as we can see on Figure 2 (a_1=0,8 and a_2=1,5 on the diagram).

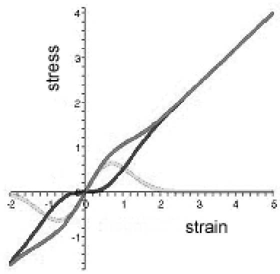

Figure 2: Stress-strain diagram by Function 1

3 THE INTERACTION OF THE TWO YARN DIRECTIONS

The yarns are woven, they make each other curved. Because of the elongation they try to be strait and try to make more curved the orthogonal direction. In one directional loading the length of the perpendicular direction decrease fast at small strain level: the opposite yarn becomes curved dramatically. At longer strain level, when the opposite yarns are already curved the perpendicular direction decrease slower. Because of this the typical horizontal section at 0 strain level of a stress-strain-strain surface function is like in Figure 3. This curve can be described very well with the following function:

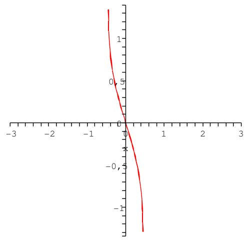

Figure 3: The horizontal section of the stress-warp strain-weft strain diagram

$$\sigma_w = 0 = c_1\varepsilon_f\left(1-\tanh\left(-\varepsilon_f\varepsilon_w\right)\right) \tag{2}$$

where ε_w and ε_f are the elongation in the weft and the fill direction and c_1 is a parameter.

4 THE STRESS STRAIN FUNCTION

From the two previous Equations (Function 1 and 2) the stress-strain function is the next:

$$\sigma_w = a_1\varepsilon_w\left(1-e^{-\varepsilon_w^2}\right)+a_2e^{-\varepsilon_w^2}+c_1\varepsilon_f\left(1-\tanh\left(-\varepsilon_f\varepsilon_w\right)\right) \tag{3}$$

and

$$\sigma_f = b_1\varepsilon_f\left(1-e^{-\varepsilon_f^2}\right)+b_2e^{-\varepsilon_f^2}+c_1\varepsilon_w\left(1-\tanh\left(-\varepsilon_f\varepsilon_w\right)\right) \tag{4}$$

.These functions represent a realistic stress-strain surface at 100% strain level, which is not realistic. In the service load level the strain is around 1-3%. The exponential parts of the functions need additional parameters to scale the functions:

$$\sigma_w = a_1\varepsilon_w\left(1-e^{-(a_3e_w)^2}\right)+a_2e^{-(a_4\varepsilon_w)^2}+c_1\varepsilon_f\left(1-\tanh\left(-c_2\varepsilon_f\varepsilon_w+c_3\right)\right) \tag{5}$$

and

$$\sigma_f = b_1\varepsilon_w\left(1-e^{-(b_3e_f)^2}\right)+b_2e^{-(b_4\varepsilon_f)^2}+c_1\varepsilon_w\left(1-\tanh\left(-c_2\varepsilon_f\varepsilon_w+c_3\right)\right) \tag{6}$$

These stress-strain functions satisfy the energy conservation criteria:

$$\frac{\partial\sigma_w}{\partial\varepsilon_f}=\frac{\partial\sigma_f}{\partial\varepsilon_w}=c_1\left[1+\frac{\varepsilon_w\varepsilon_f}{\cosh\left(-c_2\varepsilon_w\varepsilon_f+c_3\right)}-\tanh\left(-c_2\varepsilon_w\varepsilon_f+c_3\right)\right] \tag{7}$$

5 THE PARAMETER IDENTIFICATION

The parameters of the functions can be found by least square method from measured data. The parameters of the two stress functions need to be optimized together because of the interaction of the two yarn direction:

$$\Omega = \sum_{i=0}^{n} \left[\left(Q_{wi} - \sigma_{w\left(\varepsilon_i^w, \varepsilon_i^f\right)} \right)^2 + \left(Q_{fi} - \sigma_{f\left(\varepsilon_i^w, \varepsilon_i^f\right)} \right)^2 \right] \Rightarrow \min \tag{8}$$

where Ω is the function to minimize, Q_{wi} and Q_{fi} are the stress values from measurements, σ_w and σ_f are the calculated stress values for ε_{wi} and ε_{fi} strain level.

To find the minimum of Ω is simple, if a_3, a_4, b_3, b_4, c_2 and c_3 are fixed parameters. A linear system of equation needs to be solved. A practical value can be 100 for a_3, a_4, b_3, b_4, 10000 for c_2 and 0 for c_3. To optimize all the parameters the genetic algorithm can be a good suggestion.

7 CONCLUSIONS

- A new material law is described in this paper. This material law can handle both the nonlinear behavior of the yarns and the geometric nonlinearity of the fabric. Asymptotic exponential functions are used to avoid the divergence of the nonlinear structural analysis.
- The phenomenological material law allows a simple use with low computation cost. The parameter identification is simple, if the a_3, a_4, b_3, b_4, c_2 and c_3 are given parameter. If they are unknown, the identification can be very complicated.

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